Coupling of analytical and numerical methods for the electromagnetic simulation of permanent magnet synchronous machines

M. Schöning and K. Hameyer
Institute of Electrical Machines, RWTH Aachen University, Aachen, Germany

Abstract

Purpose – To reduce the computational costs for electromagnetic simulations of permanent magnet synchronous machines maintaining a high accuracy.

Design/methodology/approach – An analytical model is introduced regarding multiple designs of permanent magnet synchronous machines. This electromagnetic model is coupled to a numerical simulation. Thereby, the advantages of both computational methods are combined by parameterizing the analytical model to the numerical solution. This results in a high-efficient analytical model with the accuracy of the numerical simulation. The results of the analytical model are compared to measurements of a permanent magnet synchronous machine. Various machine modifications are simulated to evaluate possible limitations of the analytical model.

Findings – It can be stated, that a once parameterized analytical model achieves a high accuracy. Furthermore, geometric variations can be applied without the need of a new parameterization through a numerical simulation. Only changing the permanent magnet height or the air gap height results in a significant deviation and a new numerical simulation is recommended.

Research limitations/implications – Only measurements for machines up to 5 kW were available. In consequence, the model is only validatet in this range.

Practical implications – With the presented analytical model, an electromagnetic design of a permanent magnet synchronous machine can be performed very time efficient achieving accurate results. Furthermore, optimization studies can be performed with low-computational costs.

Originality/value – The introduced analytical model can be parameterized by a numerical simulation. The numeric simulation process and the parameterization are performed automatically according to the data calculated by the analytical model. Measurements demonstrate the effectiveness and the limitations of the model.

Keywords Electric machines, Simulation, Finite element analysis, Electromagnetism

Paper type Research paper

1. Introduction

The design process of electrical machines is an iterative process. Optimization algorithms can be used to automate the design process partially. Often this optimization is performed by means of appropriate algorithms, running in a program loop. Depending on design goals, the optimization can be very time consuming. Using numerical methods can result in a huge computation time. On the other hand, analytical models would probably not achieve the accuracy requirements. Here, an analytical model is introduced, which can be parameterized by the numerical simulations. An evaluation is demonstrated to determine possible limitations of the parameterized analytical model.

COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering
Vol. 27 No. 1, 2008
pp. 85-94

0332-1649
DOI 10.1108/03321640810836663

2. Analytical model

Field of application of the analytical model is the design and re-calculation process of permanent magnet synchronous machines. The purpose of the developed analytical model is the automated computation of various types of permanent magnet synchronous machines. Therefore, various types of constructions are considered. This includes the winding type, the slot shape and the attachment of the magnets. Distributed and concentrated windings, five different slot shapes as well as surface magnets or buried magnets are implemented. Owing to non-existent rotor windings and absence of damper circuits only the voltage equations and flux linkage of the stator have to be considered. The constant rotor flux Ψ_M in the direct axis, evoked by the permanent magnets, can be represented by a constant excitation current i'_{FO}. The following set of dynamic equations in the dq-reference frame is obtained (Vas, 1990):

$$v_{\mathrm{d}} = R_1 \cdot i_{\mathrm{d}} + \frac{\mathrm{d}\Psi_{\mathrm{d}}}{\mathrm{d}t} - \omega \cdot \Psi_{\mathrm{q}}, \tag{1}$$

$$v_{\mathrm{q}} = R_1 \cdot i_{\mathrm{q}} + \frac{\mathrm{d}\Psi_{\mathrm{q}}}{\mathrm{d}t} - \omega \cdot \Psi_{\mathrm{d}}, \tag{2}$$

$$T_{\mathrm{el}} = p \cdot (\Psi_{\mathrm{d}} \cdot i_{\mathrm{q}} - \Psi_{\mathrm{q}} \cdot i_{\mathrm{d}}) = \frac{J_{\mathrm{m}}}{p} \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} + T_{\omega}, \tag{3}$$

$$\Psi_{\mathrm{d}} = L_{\mathrm{d}} \cdot i_{\mathrm{d}} + \underbrace{L_{\mathrm{hd}} \cdot i'_{\mathrm{FO}}}_{\Psi_M}, \tag{4}$$

$$\Psi_{\mathrm{q}} = L_{\mathrm{q}} \cdot i_{\mathrm{q}}. \tag{5}$$

Ψ represents the flux linkage, T the torque, v the voltage, R_1 the stator resistant, L_{d} the inductance in direct axis, L_{q} the inductance in quadrature axis and J_{m} the moment of inertia. For steady-state operation, $\mathrm{d}\Psi/\mathrm{d}t$ and $\mathrm{d}\omega/\mathrm{d}t$ are zero. After re-transforming from the rotating coordinate system, used to yield constant mutual reactances and with the assumption $I_{\mathrm{d}} = 0$, the following set of steady-state equations is obtained:

$$V_{\mathrm{d}} = -X_{\mathrm{q}} \cdot I_{\mathrm{q}}, \tag{6}$$

$$V_{\mathrm{q}} = R_1 \cdot I_{\mathrm{q}} + V_{\mathrm{p}}, \tag{7}$$

$$T = \frac{3 \cdot p}{\omega} \cdot V_{\mathrm{p}} \cdot I_{\mathrm{p}}. \tag{8}$$

These are the main equations, used for the analytical simulation.

For the design of a new machine, the winding type, the slot shape and the attachment of the magnets as well as the parameters collected in Table I have to be known. In the first step, the inner-stator diameter is calculated using the D^3L approach (Honsinger, 1987):

$$D_0 = \sqrt[3]{p \cdot Pr \cdot \frac{(1 + aK)}{\mathrm{eta\_}r \cdot pK \cdot lK \cdot K\omega \cdot B_{\mathrm{g}}M \cdot cA \cdot nr \cdot \pi^2}}. \tag{9}$$

To verify the first computation, the factor *kpow*, defined by:

$$kpow = \frac{Pr}{Pol}, \tag{10}$$

p		Number of Pol Pairs
P_r	(W)	Rated output power
aK		Electric loading coefficient
$eta\text{-}r$		Efficiency
pK		Cosinus of half phase angel
lK		Core stack length – inner stator diameter ratio
Kw		Winding factor
BgM	(T)	Given middle air-gap induction
cA	(A/m)	Electric loading in the stator
nr	(I/s)	Rated speed
ag	(m)	Given air-gap height

Table I. Necessary parameters for diameter calculation

is determined. This factor represents, the relation between the assumed power Pr and the actually generated power $Po1$. The illustrated case differentiation (Figure 1) regulates the factor to be in a range between 0.8 and 1.1. Further, details of the inner-stator diameter calculation are shown in Figure 1. In the next step, the stator is dimensioned (Figure 4).

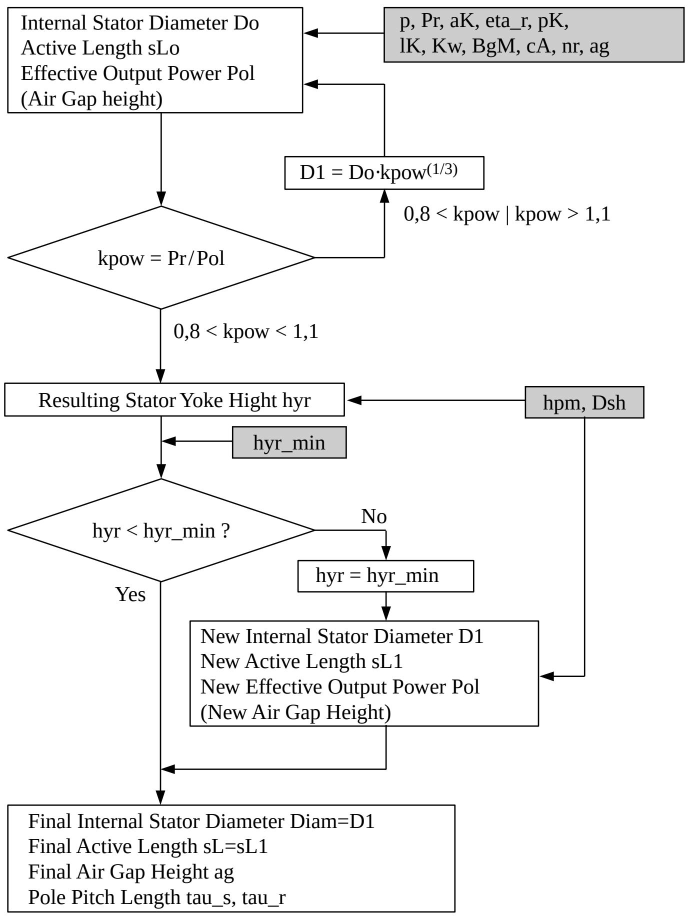

Figure 1. Determination of inner-stator diameter

Depending on the selected winding type, the winding factor is calculated. Afterwards, the windings per phase and the necessary winding area is computed regarding the power, the winding layers and the air gap flux density. With this values, the diameter of the conductors and the coil area can be determined. With the knowledge of the conductor diameter, the slot dimensions are evaluated. A case differentiation is made to regard trape-zoidal and rectangular slot shapes. Available slot shapes are shown in Figures 2 and 3. At this point, all necessary stator geometries are known and the needed electrical values like flux leakage, reactances and the carter factor can be calculated (Figure 4). When the electrical values of the stator are known, the rotor dimensions are determined. Here, a case differentiation is made to regard surface mounted or buried magnets.

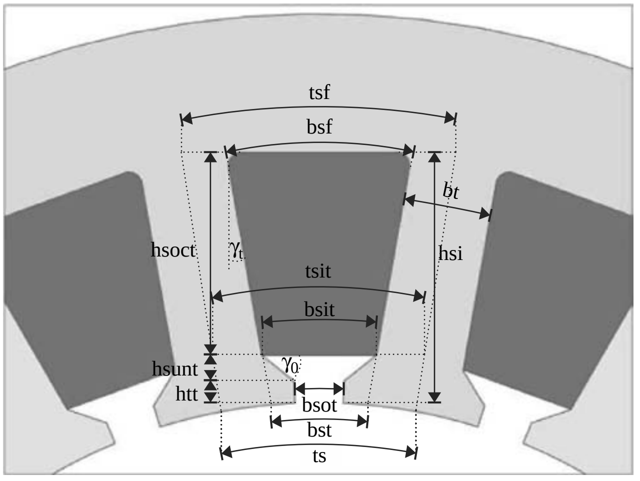

Figure 2.
Trapezoidal slot shapes

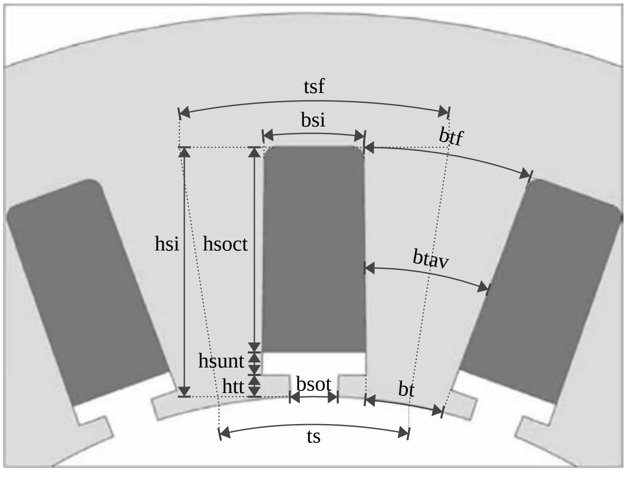

Figure 3.
Rectangular slot shapes

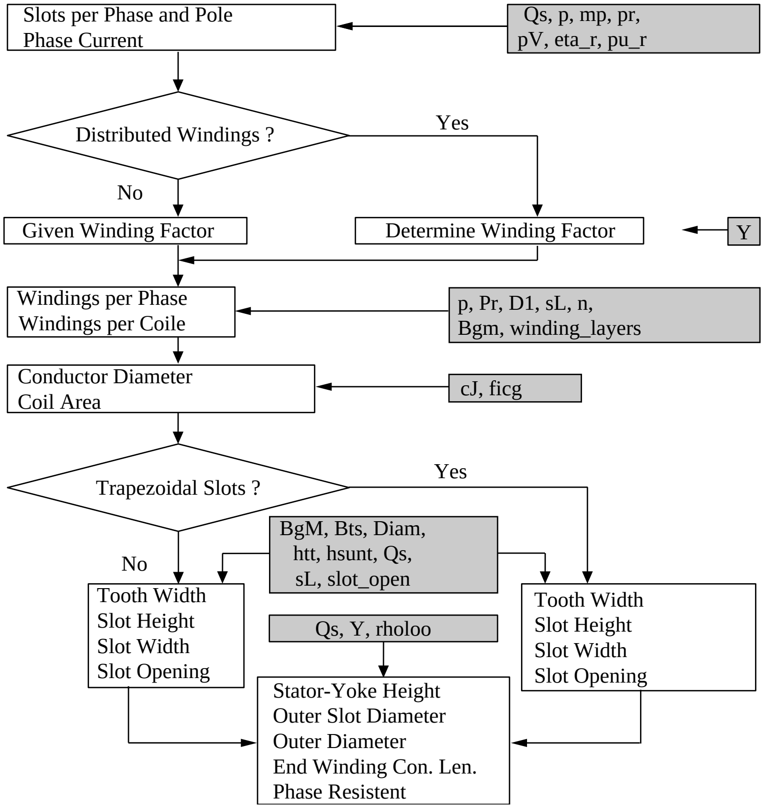

Figure 4.
Stator values

Afterwards, a two dimensional numerical model can be built automatically by transferring the required geometry parameters from the analytical model. In the last step of the analytical model, the main electrical characteristics are calculated (Figure 5). With theses characteristics, the problem definition for the numerical solver can be created.

3. Numerical simulation

The numerical simulation is performed using iMOOSE (Riesen *et al.*, 2004). This software package consists of solvers for different problem definitions such as static, transient and harmonic problems. For this study the node-based transient FEM solver for 2D electro-magnetic field problems with eddy-current regions is employed. Owing to the requirements of an automated-design process, the problem definition and the mesh must be generated automatically, depending on the input parameters of the examined machine. For the mesh generation, Gmsh (Geuzaine and Remacle, 2006) is employed controlled by a script, which is parametrized through the output of the analytical model. Examples of the resulting geometry and mesh are shown in Figures 6 and 7. The figures show a 90° cut out of the geometry and the mesh, respectively. A complete 360° model is simulated due to the requirement, that the numerical model has to work independent of the number of Pol Pairs and slots. The results required for parameterising the analytical model are the induced

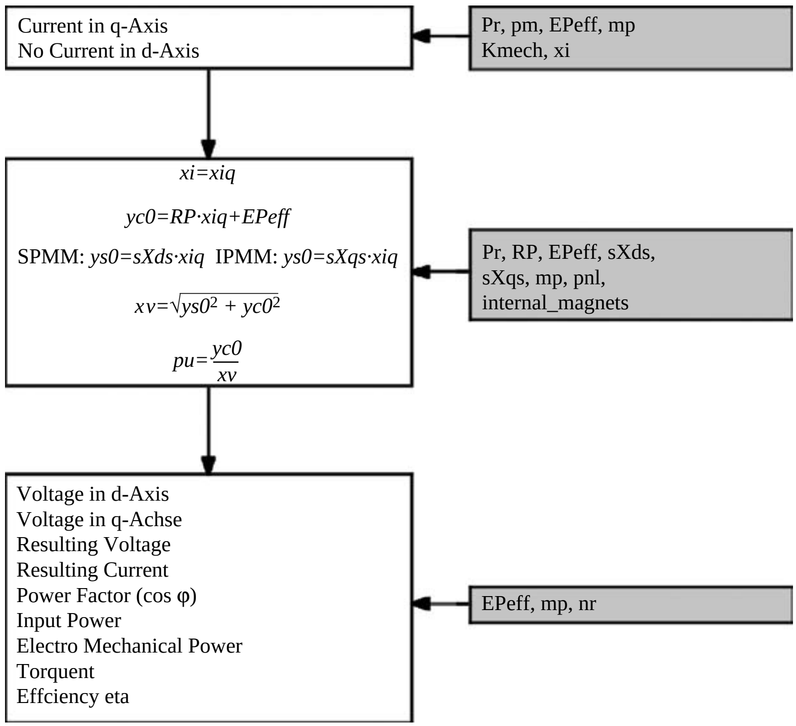

Figure 5.
Main electrical characteristics

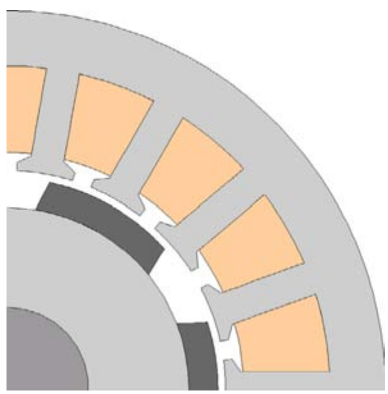

Figure 6.
Geometry

voltage, the torque and the air-gap flux density. These results are calculated automatically regarding the motor geometry and the winding distribution. The induced voltage and the flux are shown in Figure 8 exemplarily.

4. Coupling

The main electromagnetic characteristics of an electrical machine are the torque T, the mechanical power P_m and the induced voltage V_i. Owing to their importance, these

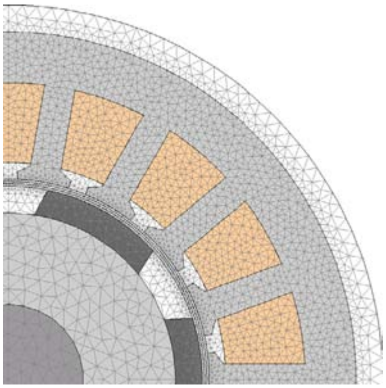

Figure 7.
Mesh

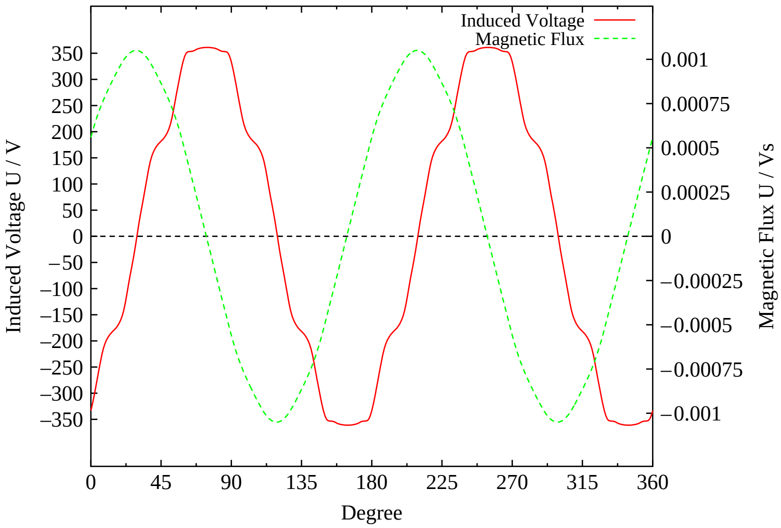

Figure 8.
Induced voltage and flux

values need to be calculated very accurately by the analytical model. The torque is defined as:

$$T = \frac{P_\mathrm{m}}{2 \cdot \pi \cdot n} \tag{11}$$

For a fixed working point the speed is constant. Therefore, the torque depends on the mechanical power only defined as:

$$P_m = \mathrm{m}p \cdot I_\mathrm{Phase} \cdot V_\mathrm{i}. \tag{12}$$

For the induced voltage, the following equation is obtained:

$$V_\mathrm{i} = \frac{1}{\sqrt{2}} \cdot w_\mathrm{Phase} \cdot Kw \cdot \omega \cdot \frac{BgMc \cdot Diam \cdot sl}{p}, \tag{13}$$

where, ω represents the angular frequency of the stator, $BgMc$ the maximal air gap flux density and w_{Phase} the windings per phase. All mentioned values in equation (13) depend on fixed values like the geometry of the machine or the number of windings and their distribution, except the maximum air gap flux density. The equation is:

$$BgMc = \sqrt{2} \cdot \mathrm{Bpmc} \cdot \frac{A_{\mathrm{pole}}}{\mathrm{A_{gc}}} \cdot \frac{1}{K\phi_\sigma} \cdot KBgMc \quad (14)$$

Bpmc represents the effective magnet flux density, calculated by the remanent flux density and the magnet area. A_{pole} describes the magnet area per pole and A_{gc} the magnetized area in the stator. The factor ϕ_σ represents the leakage flux of the permanent magnets. The parameter K BgMc is introduced to couple the analytical and numerical models. It adjusts the air gap flux density and therefore, the induced voltage, the mechanical power and the torque as well. A schematic representation of the coupling process is shown in Figure 9. This figure shows the design-process of a permanent magnet synchronous machine. First, the inner stator diameter is calculated. The coil and slot dimensions are evaluated. After completing the dimensioning of the rotor, the air gap flux density and the induced voltage is computed. With this values, the final computation is performed, by which the torque and the power is obtained. After this step,

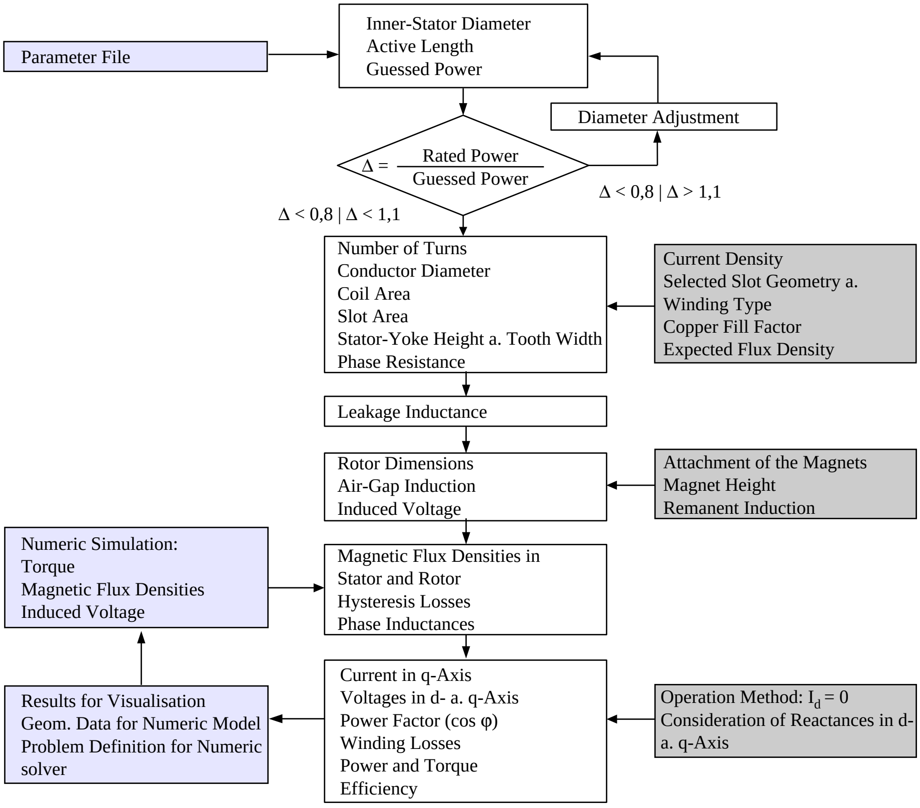

Figure 9.
Coupling analytical and numerical model

the first analytical calculation is terminated. At this point, the problem definition for the numerical-solution process is generated. In addition to, the geometrical data, directly imported from the main parameter file, the analytical calculated current, the slot cross section and the number of slots is transferred to the numerical solver. Therewith, the problem definition is complete and the numerical simulation can be started. Afterwards, the numerical calculated torque and induced voltage is extracted to adjust the coupling factor $KBgMc$ in the analytical model automatically.

5. Modifications

The analytical and numerical results are compared to the measurements of a known machine. For the machine studied, the deviation between the numerical computed torque and the measured torque is less than 1 percent. Because the torque is an initial condition for the analytical model, only the induced voltage and the geometry differs from the known machine. These deviations can be reduced to less than 1 percent by parametrizing through the numerical simulation. To reduce the overall computation time, the numerical simulation is only performed when the accuracy of the analytical model degrades significantly. To evaluate the parameters, that affect the accuracy, the analytical model is parametrized once by the numerical simulation. Afterwards, various modifications are applied to the geometry, the magnets and the current. These modifications are calculated analytically as well as numerically and the results are compared. In Figure 10, the deviation of the computed torque is plotted. Negative values indicate, that the analytical result is smaller compared to the numerically obtained result. The first modification halves the current in the stator coils. The analytical model calculates a 2 percent smaller torque then the numerical simulation. When adjusting the current to 2/3 of the nominal value, the derivation is −1.8 percent. The difference increases up to +1.6 per cent when the current is increased to three times of its nominal value. The maximum deviation is obtained when the magnet height is varied. Altering the magnet height Mh_N by 1.6 results in a difference of +7.65 percent. Varying the pole-pitch P_{bd}, the stator-yoke height Syh_N or the tooth width Tw_N yield a small deviation of less then 2.2 percent. It is obvious, that most geometry variations can be calculated with a once parameterized analytical model obtaining accurate results. There is no need for further numerical simulations.

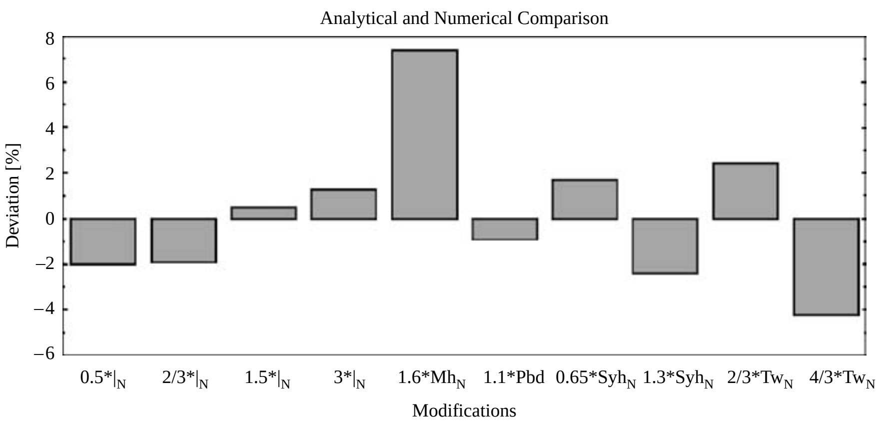

Figure 10. Comparison of analytical and numerical results

Only varying the air gap height, respectively, the magnet height for a rotor consisting of surface mounted magnets leads to less accurate results. In this case, a new numerical simulation should be performed to adjust the analytical model again.

6. Conclusions

In this paper, a coupling between analytical and numerical calculation methods is shown. The coupling allows a significant increase of accuracy for the analytical computation, whereas the computation time compared to typical numerical simulations is decreased. The combination of analytical and numerical simulation processes are able to decrease the computation time of typical optimization algorithms significantly. Especially, because iterations can be calculated exclusivley with the once parameterized analytical model. Furthermore, one generalised analytical model can be used for a wide range of permanent magnet synchronous machines by parameterizing through numerical simulations. This offers the possibility to develop an autonomously working-design process for permanent magnet synchronous machines.

References

Geuzaine, C. and Remacle, J.F. (2006), "Gmsh: a three-dimensional finite element mesh generator with built-in pre-and post-processing facilities", available at: www.geuz.org/gmsh/

Honsinger, V.B. (1987), "Sizing equations for electrical machinery", *IEEE Transactions on Energy Conversion*, Vol. EC-2 No. 1.

Riesen, D., van Monzel, C., Kaehler, C., Schlensok, C. and Henneberger, G. (2004), "iMOOSE – an open-source environment for finite-element calculations", *IEEE Transactions on Magnetics*, Vol. 40 No. 2, pp. 1390-3.

Vas, P. (1990), *Vector Control of AC Machines*, Clarendon Press, Oxford.

About the authors

M. Schöning received his MSc from RWTH Aachen University in 2004. Currently he is PhD student at the Institute of Electrical Machines of RWTH Aachen University. His fields of interests are virtual reality techniques as well as the development of analytical and numerical design tools for electrical machines. M. Schöning is the corresponding author and can be contacted at: marc.schoening@iem.rwth-aachen.de

K. Hameyer received his PhD from the University of Technology Berlin, Germany, 1992. From 1986 to 1988 he worked with the Robert Bosch GmbH in Stuttgart, Germany, as a Design Engineer for Permanent Magnet Servo Motors. In 1988, he became a member of the staff at the University of Technology Berlin, Germany. Until 2004, he was Professor of numerical field computation and electrical machines with the K.U. Leuven and a Senior Researcher with the FWO-V in Belgium, teaching CAD in Electrical Engineering and Electrical Machines. Currently, he is the Head of the Institute of Electrical Machines of RWTH Aachen University, Germany. His research interests are numerical field computation, the design of electrical machines, in particular permanent magnet excited machines, induction machines and numerical optimization strategies. He is a member of the International Compumag Society and senior member of the IEEE. E-mail: kay.hameyer@iem.rwth-aachen.de