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# A dynamical vector hysteresis model based on an energy approach

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***Abstract*—A dynamical vector hysteresis model is presented, which is a generalisation of a quasi-static model proposed in a recent paper. The model can be considered from the point of view of a mechanical analogy with the pinning of Bloch walls phenomenon represented by a friction force. By combining several elementary submodels with each other, the number of parameter can be increased for a better accuracy.**

## I. Introduction

The quality of hysteresis models is generally assessed on basis of their ability to reproduce accurately magnetic *b-h* curves obtained from measurements. But, if one is interested in the computation of losses or forces in a magnetic material with hysteresis, the ability of matching *b-h* curves is no longer a sufficient proof of the quality of the model. On the contrary, a complete material model is required, which is able to provide the different terms of the local energy balance in the material and from which the constitutive laws can be derived consistently.

The model of Preisach [1], for instance, has no real interpretation in terms of energy, and further assumptions are required if one wants to use it in coupled problems [2][3]. On the other hand, the basic assumptions of the Jiles-Atherton model [4] constitute a true material model with an interpretation in terms of energy. However, at a certain point in the development of the model, algebraic and differential operations are performed, which make loose track of the grounding energy concepts. At the end, the model does not generalise naturally to 2 or 3 dimensions of space. Nor provides it an energy balance of the material.

In a recent paper [5], an alternative hysteresis model has been proposed, which is based on the same basic assumptions as the Jiles-Atherton model, but remains all through consistent with the energy interpretation. As it is obtained directly from a complete material model, this hysteresis models is readily a vector model, and it does not need to be explicitly vectorized. In this paper, a dynamical term is added so as to obtain a dynamical vector model. The issue of the identitifation of the free parameters of the model is also addressed.

## II. Energy balance

The proposed dynamic hysteresis model can be considered from the point of view of the following mechanical analogy. Consider a object free to slide on a rough surface and attached with a spring to a fixed point $P$. The whole system is plunged in a viscous liquid. The object is subjected to a known external force $\mathbf{h}$, to the restoring force

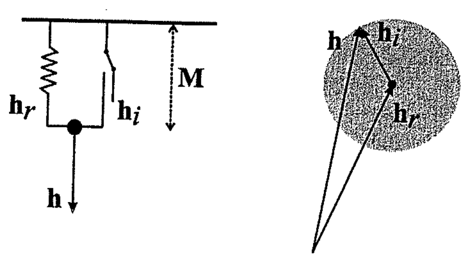


Fig. 1. Mechanical analogy and pictorial representation of the vector model. The grey circle represents the subgradient $G$.

$-\mathbf{h}_r$ of the spring and to a dissipative force $-\mathbf{h}_i$, which is due to the friction on the plane and to the viscosity of the fluid. The position of the object is denoted by the vector $\mathbf{M}$, $\dot{\mathbf{M}}$ is the velocity. Inertia is neglected.

The differential equation ruling this system can be obtained by a functional approach based on the the first law of Thermodynamics $\dot{\rho} = \dot{\rho}^W + \dot{\rho}^Q$, here written in terms of power densities. The internal energy of this system is the energy stored in the spring, which is a (singled valued) function $\rho : \mathbf{M} \mapsto \mathbb{R}$. The power developed by the external force $\mathbf{h}$ writes $\dot{\rho}^W = \mathbf{h} \cdot \dot{\mathbf{M}}$. One has

$$\dot{\rho}(\mathbf{M}) = \mathbf{h} \cdot \dot{\mathbf{M}} - \kappa\,|\dot{\mathbf{M}}| - \lambda \dot{\mathbf{M}}^2, \tag{1}$$

where the last two terms represent respectively the dissipation due to friction and the dissipation due to viscosity.

As this relation must be verified at any time whatever the trajectory of the object, the equilibrium equation is found by factorizing $\dot{\mathbf{M}}$. If $\rho$ is assumed to be differentiable with respect to $\mathbf{M}$, one has at the left hand side

$$\dot{\rho}(\mathbf{M}) = \mathbf{h}_r \cdot \dot{\mathbf{M}} \quad \text{with} \quad \mathbf{h}_r = \partial_{\mathbf{M}}\rho(\mathbf{M}). \tag{2}$$

At the right hand side, however, the dissipation functional $\dot{\rho}^Q = \kappa|\dot{\mathbf{M}}| + \lambda\dot{\mathbf{M}}$ is not differentiable at $\dot{\mathbf{M}} = 0$, due to the presence of the $|\dot{\mathbf{M}}|$ term. But, as it is convex, it has a subgradient $G$ defined by

$$G = \{\mathbf{h}_i, |\mathbf{h}_i| \leq \kappa \text{ if } \dot{\mathbf{M}} = 0, \mathbf{h}_i = \kappa\, \mathbf{e}_{\dot{\mathbf{M}}} + \lambda\dot{\mathbf{M}} \text{ if } \dot{\mathbf{M}} \neq 0\}. \tag{3}$$

with $\mathbf{e}_{\mathbf{X}} \equiv \mathbf{X}/|\mathbf{X}|$, and represented by the grey circle in Fig. 1. The equilibrium equation writes now

$$\mathbf{h} - \mathbf{h}_r = \mathbf{h}_i \in G. \tag{4}$$

The memory effect originates from the non-differentiable character of the functional $|\dot{\mathbf{M}}|$, as the latter implies the non-univocity of the friction force $\mathbf{h}_i$. The subgradient is indeed a whole set of possible gradients (i.e. of possible forces), whereas a differentiable functional has one and only one gradient at each point. If $\mathbf{h}$ is inside the circle, one has $\dot{\mathbf{M}} = 0$, which implies $\dot{\mathbf{h}}_r = 0$: The elongation of the spring does not change. In this way, the non-univocity of $\mathbf{h}_i$ makes it possible to maintain a given $\mathbf{h}_r$, and hence a given magnetisation $\mathbf{M}$, even when the magnetic field $\mathbf{h}$ has yet decreased, whence the memory effect.

## III. Analogy

This mechanical model can be identically translated into an analogous model for ferromagnetic materials with hysteresis. The vector $\mathbf{M}$ is now the magnetisation of the material (in Tesla). The applied force $\mathbf{h}$ is the magnetic field. The friction force $\mathbf{h}_i$ originates from the pinning of Bloch walls phenomenon (main cause of the magnetic hysteresis) and from the viscosity force due to the induction of eddy currents in the material when the magnetisation varies in time. Both dissipative forces have the dimensions of a magnetic field.

Similarly to the Jiles-Atherton model, the magnetisation $\mathbf{M}$ is obtained by the Langevin model

$$\mathbf{M}(\mathbf{h}_r) = M_s L\left(\frac{|\mathbf{h}_r|}{h_o}\right)\mathbf{e}_{\mathbf{h}_r} \quad , \quad L(x) = \coth x - \frac{1}{x}, \quad (5)$$

and the induction is $\mathbf{b}(\mathbf{h}) = \mathbf{M}(\mathbf{h}_r) + \mu_0\mathbf{h}$. The dissipated power is $\mathbf{h}_i \cdot \dot{\mathbf{M}}$. Unlike the Jiles-Atherton model, for which the magnetisation is decomposed into a reversible and an irreversible part, the applied field $\mathbf{h}$ is in this model decomposed into the reversible part $\mathbf{h}_r$ (nonlinear spring force) and the irreversible part $\mathbf{h}_i$, Fig. 1.

## IV. Implementation

Like any hysteresis models, this model fits naturally into a magnetic field formulation. Knowing $\mathbf{h}$ and the definition (3) of the subgradient, which is the set of possible values for $\mathbf{h}_i$, different update rules for $\mathbf{M}$ can be devised, with different level of accuracy, as detailed in the full paper. The non-differentiable character of the dissipation functional is only a theoretical problem. It amounts to a simple test in the implementation. With first order shape function, the unknown field $\mathbf{h}$ is constant in each element and the hysteresis algorithm requires to store the value of the vector $\mathbf{M}$ for each ferromagnetic element. As the update rule is a vector relation, it gives as such a vector hysteresis model, without making any other assumption.

## V. Combined model

In the elementary form presented so far, the model has only four parameters: $M_s$ and $h_o$ to represent the anhysteretic curve, $\kappa$ and $\lambda$ to represent hysteresis. Although it gives yet qualitatively interesting results for the main hysteresis loop, a better representation of the material behaviour requires to dispose of a larger number of free parameter. This can be achieved as described in [5]. Fig. 2 shows the match obtained between the model and quasistatic measurements.

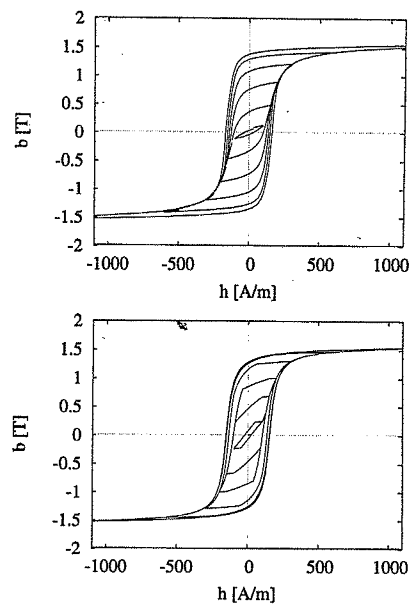


Fig. 2. Measurements (above) and model (below) with 5 fractions for electrical steel.

## Conclusion

Unlike the models of Preisach and Jiles-Atherton, this model is readily a vector model and the differents terms of the local energy balance of the material (stored magnetic energy, dissipated energy) are known at all times. Unlike the model of Jiles-Atherton, the number of parameters is not limited. The combined model offers an arbitrary number of parameters. However, due to internal constraints in the model, not all hysteresis curves can be matched exactly. Such a limitation is comparable with the congruence property of the Preisach model [1]. A material model for a magnetostrictive material with hysteresis could so be obtained directly by substituting to $\rho$ a magnetostrictive functional.

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