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Skew-Discretization Error for Structural-Dynamic Finite-Element Simulations of Electrical Machines

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***Abstract*—2D FE models of skewed electrical machines apply the Multi-Slice Method (MSM). There are various publications dealing with MSM concerning electromagnetic simulations and their accuracy. Here, an error estimation is transformed to a structural-dynamic model using 2D surface-force excitation.**

I. Introduction

The acoustic noise dissipated by electrical machinery is attracting more and more attention. As an electric power-steering drive an Induction Machine with squirrel-cage rotor (IM) has been implemented. Large effort has been focused on the structural-dynamic simulation of this IM. In the electromagnetic domain the IM needs to be simulated using 2D transient FE simulation. The skew of the IM also affects the surface-force density-excitation of the structural-dynamic model. Therefore, the error of transforming the forces from the 2D electromagnetic MSM model is an object of investigation. In this paper the Skew-Discretization Error (SDE) introduced in [1] is applied to the surface-force density-transformation of the mechanical model. Now, new transformation schemes can be derived reducing the mechanical SDE.

Three different slice positions and weights for the MSM are regarded varying the number of slices n_{sl}. The slices of the uniform discretization (UD) are positioned equidistantly and have the same weight. The edge-uniform discretization (EUD) has one slice each on the front and rear of the machine and uniformly distributed slices in between. The Gauß discretization (GD) also considered, becomes more dense to the edges.

II. Skew-Discretization Error

As [1] describes the classical rotating-field theory defines a skew using skew factors F_{sk}. An induction-field wave B seen by the stator of the spatial order p and depending on the angular frequency ω and the skewing angle Θ_{sk} reads:

$$\begin{aligned}\frac{1}{\Theta_{sk}}\int_{-\Theta_{sk}/2}^{\Theta_{sk}/2} B\cos(p\Theta_0 + p\Theta'_{sk} + \omega t)\ d\Theta'_{sk} \\ = BF_{sk}\cos(p\Theta_0 + \omega t)\,.\end{aligned} \quad (1)$$

Θ_0 is the initial mechanical angle. The skew factor for the continuous flux-density description results in

$$F_{sk} = \frac{\sin\phi}{\phi} \qquad \text{with} \quad \phi = \frac{p\Theta_{sk}}{2}\,. \quad (2)$$

For the MSM the skew is discretized in n_{sl} slices of the machine defined by their position η_i and their weight γ_i. The integral from equation (1) becomes a sum:

$$\begin{aligned}\frac{1}{\Theta_{sk}}\sum_{i=1}^{n_{sl}} B\cos(p\Theta_0 + p\gamma_i\frac{\Theta_{sk}}{2} + \omega t)\cdot\eta_i\Theta_{sk} \\ = BF_{sk,n_{sl}}\cos(p\Theta_0 + \omega t)\,,\end{aligned} \quad (3)$$

and the skew factor for discrete skewing results in:

$$F_{sk,n_{sl}} = \sum_{i=1}^{n_{sl}} \eta_i\cos(\gamma_i\frac{p\Theta_{sk}}{2})\,. \quad (4)$$

The SDE is now easily estimated by:

$$\text{SDE} = \frac{F_{sk,n_sl} - F_{sk}}{F_{sk}}\,. \quad (5)$$

III. The SDE of the Air-Gap Flux-Density

The IM studied in this paper has $p = 2$ pole pairs, $N_S = 36$ stator and $N_R = 26$ rotor slots respectively. For the SDE estimation, the flux-density behavior along

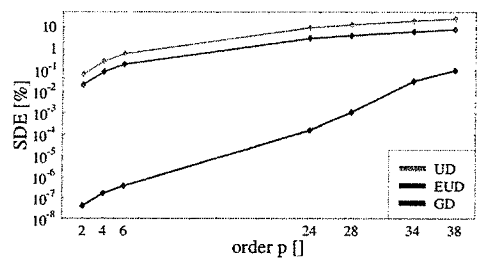

Fig. 1. Absolute Values of the SDE for 5 Slices.

a circle in the air-gap is derived from one time step of the 2D transient FE simulation. The main orders obtained from Fourier's transformation are the 2^{nd}, 24^{th}, 28^{th}, 34^{th}, 38^{th}. The 2^{nd} is responsible for the load torque of the machine. The others produce alternating torque contributions. For the structural dynamic model, to which the SDE is applied, only low orders $r \leq 8$ are of interest. Higher orders do not result in significant deformation. Therefore, the 4^{th} and 6^{th} order are studied as well. The SDEs for 5 slices are depicted in Fig. 1. The GD results in the most accurate results affording less slices than UD and EUD.

IV. Surface-Force Density-Error

From the flux-density distribution the surface-force density σ on the stator teeth is derived applying the Maxwell-stress tensor. The surface-force density is transformed to a 3D mechanical model consisting of the entire machine geometry. From each slice of the MSM model the surface-force density is transformed to a certain region of the mechanical model, which has 14 element layers in axial direction. This leads to overlapping slice ranges if 5 slices are used in the electromagnetic MSM model. The values of both ranges are weighted and averaged. Fig. 2 demonstrates this procedure. The element layer 2 of the

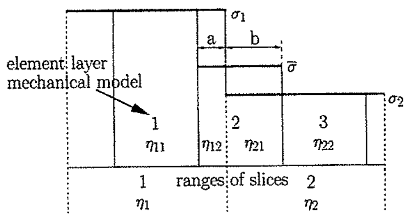

Fig. 2. Weighting and Averaging of the Force Values.

mechanical model corresponds to the ranges 1 and 2 of the electromagnetic MSM model:

$$\overline{\sigma} = \frac{a\sigma_1 + b\sigma_2}{a+b} = \frac{\eta_{12}\sigma_1 + \eta 21\sigma_2}{\eta_{12} + \eta_{21}} \tag{6}$$

with $a = \eta_{12} \cdot l_{z,2}$ and $b = \eta_{21} \cdot l_{z,2}$. $l_{z,2}$ is the length of the element layer 2 EL_2. The addend for EL_2 in (3) replacing B by σ now reads:

$$\frac{1}{\Theta_{sk}} \cdot \int_{EL_2} \sigma \, d\Theta'_{sk} \quad = \quad \frac{1}{\Theta_{sk}} \cdot \left(\overline{\sigma} \cdot (\eta_{12} + \eta_{21})\, \Theta_{sk}\right). \tag{7}$$

For the non-averaged skew equation (3) becomes:

$$\begin{aligned} \frac{1}{\Theta_{sk}} \cdot \int_{EL_2} \sigma \, d\Theta'_{sk} &= \frac{1}{\Theta_{sk}} \cdot \hat{\sigma}\cos(\epsilon) \cdot \eta_{12}\Theta_{sk} \\ &+ \frac{1}{\Theta_{sk}} \cdot \hat{\sigma}\cos(\epsilon) \cdot \eta_{21}\Theta_{sk}\,, \end{aligned} \tag{8}$$

with $\epsilon = p\Theta_0 + p\gamma_{12}\frac{\Theta_{sk}}{2} + \omega t$. For $\sigma_1 = \hat{\sigma}\cos(\epsilon)$ and $\sigma_2 = \hat{\sigma}\cos(\epsilon)$ and (6), (8) is equivalent to (7). This means, that the averaging and weighting formula introduced in (6) results in the same SDE as in the case of non-overlapping electromagnetic slices for element layers in the mechanical model. Instead of the surface-force density the flux density can be averaged as well. This is closer to the physical condition that the surface-force density results from the flux-density. With $\sigma \sim B^2$ it follows

$$\overline{\sigma} \sim \underbrace{\left(\frac{aB_1 + bB_2}{a+b}\right)^2}_{b_{12}} \neq \underbrace{\frac{aB_1^2 + bB_2^2}{a+b}}_{b_1 b_2}. \tag{9}$$

The term marked with $b_1 b_2$ results from (6) if σ is replaced by B^2. The SDE for the surface-force density could be reduced by adapting this interrelationship to the MSM-to-mechanical-model transformation scheme.

V. Results

Here, results of structure-borne noise-simulations of a 3D model and the three MSM-models compared to measurements are presented. Exemplarily the first rotor-slot harmonic $r = 26 \cong 520\,\text{Hz}$ of the IM is analyzed (Fig. 3). Due to the fact that L_S is a local value and the SDE a

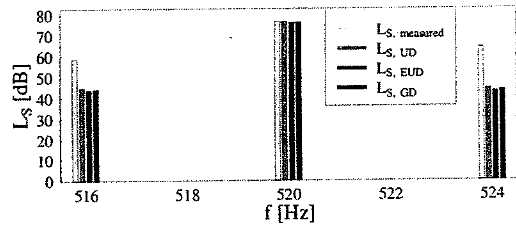

Fig. 3. Structure-Borne Noise-Level L_S at 520 Hz.

global one, marginal differences between the three MSM-models' results are stated. Nevertheless, the GD delivers the most accurate results for the electromagnetic simulation concerning the SDE and therefore should be applied.

VI. Conclusion

In this paper the SDE proposed in [1] is studied for different types of discretization and is transformed to a SDE for the surface-force density. It can be stated that the SDE for σ can be improved by transforming B first and then deriving σ from B. A further improvement will be reached by using a more sophisticated averaging of σ for a mechanical element layer which is connected to two electromagnetic slices. Further results and more aspects concerning the SDE will be presented in the full paper.

References

[1] J. J. Gyselinck, L. Vandevelde, and J. A. A. Melkebeek, "Multi-slice fe modeling of electrical machines with skewed slots – the skew discretization error," *IEEE Transactions on Magnetics*, vol. 37, pp. 3233–3237, September 2001.