Combined Numerical and Analytical Method for Geometry Optimization of a PM Motor

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***Abstract*—For cogging torque reduction of permanent-magnet synchronous-machines (PMSM) many approaches are known. But cogging-torque predictions for arbitrary types of machines are very imprecise. Therefore, a fast optimization method is desirable. The combination of numerical and analytical models results in such an approach.**

I. INTRODUCTION

The standard servo drive is a PMSM. One major task in developing PM machines is to minimize the cogging torque. The common methods can be divided into three groups. They consider the air-gap flux-density distribution, the relative air-gap permeance-function, or they compensate the cogging torque. In this paper, a combined numerical and analytical method is applied to the optimization of the stator-tooth notch-shape.

II. ANALYTICAL MODEL

Inserted notches in the tooth shape produce supplementary cogging torque, which is phase shifted due to the location of the notches [1] resulting in a lower total cogging torque. By the insertion of the notches the effective number of stator slots N_s is increased. The fundamental order of the cogging torque, which reaches the highest amplitude of all orders by far, increases for an adequate number of notches N_n. The machine studied consists of $N_p = 2p = 8$ permanent-magnet rotor-poles and $N_s = 18$ stator teeth. Therefore, the fundamental order of the cogging torque is the Least Common Multiple LCM$(18,8) = 72$. The optimal number of notches per tooth is $N_n = 2$. For the optimal notch shape the energy-method is applied. With the air-gap magnetization B_m and the relative air-gap permeance-function $\Lambda(\Theta,\alpha)$:

$$W(\Theta) = \frac{1}{2\mu_0}\int_V B_m^2(\alpha)\Lambda^2(\Theta,\alpha)\mathrm{d}V\,. \tag{1}$$

Θ is the relative position of stator and rotor and α the angular location in the air gap. The square of the flux density produced by the magnets and the square of the relative air-gap permeance-function are discomposed:

$$\Lambda^2(\theta,\alpha) = \sum_{i=0}^{\infty} a_{i\cdot N_s}\cos(i\cdot N_s(\theta+\alpha)) \tag{2}$$

$$B_m^2(\alpha) = \sum_{j=0}^{\infty} b_{j\cdot N_p}\cos(j\cdot N_p\alpha) \tag{3}$$

where $a_{i\cdot N_s}$ and $b_{j\cdot N_p}$ are the Fourier coefficients. To simplify the expression for the calculation of the cogging torque, it is considered that the relative air-gap permeance-function does not change with the radius. Thus, the cogging torque can be written as:

$$\begin{aligned} T_{cog}(\theta) = {} & \frac{l_m\pi}{4\mu_0}(R_s^2 - R_r^2)\cdot \\ & \cdot\sum_{n=0}^{\infty}\Big(n\cdot N_{CT}\cdot a_{n\cdot N_{CT}}\cdot b_{n\cdot N_{CT}}\cdot\sin(n\cdot N_{CT}\cdot\theta)\Big) \end{aligned} \tag{4}$$

where l_m is the machine length, R_s the stator radius, R_r the rotor radius and N_{CT} the least common multiple LCM$(18,8) = 72$. It is sufficient to optimize Λ^2 or B_m^2 for the most significant harmonics of the cogging torque. For example, the optimization of the studied machine can be performed by minimizing the 72^{nd} components With the assumption that Λ is rectangular its coefficients read [2]:

$$\begin{aligned} a_{nN_s} = {} & \frac{2}{\pi n}\Big(\sin\big(nN_s\frac{w_t}{2}\big) - \sin\big(nN_s(\frac{w_t}{2}+w_n)\big) \\ & + \sin\big(nN_s(\frac{3w_t}{2}+w_n)\big)\Big), \quad n\geq 1\,. \end{aligned} \tag{5}$$

Where w_n is the width of the notches, w_s the width of the slots and w_t the distance between the end of a slot or notch and the beginning of the next one. The minimum, in fact "0", is reached for $w_n = w_s$. This corresponds to the result [1] obtained. To proof the cogging-torque behavior a static 2D finite-element model is applied comparing the notched with the basic machine model without notches. Contrary to the results [1] achieved, the peak-to-peak values of the cogging torque rise strongly. Thus, the analytical model applied in [1] does not suit here and a numerical optimization is applied instead.

III. NUMERICAL MODEL

A step-by-step numerical optimization of the notch shape is performed. Parameters are the width w_n and height h_n of the rectangular notch. The cogging torque does not depend strongly on the height. The resulting load torque will be the highest for the smallest notch. Therefore, the smallest height of the notch is chosen: $h_n = 0.4h_s$. For the minimization of load-torque loss the smallest width w_n is chosen. Therefore, a finer exploration is performed. The optimal width of the notch is found to be $w_n = 0.17w_s$. This does not correspond to the analytical solution found and disproved in the previous section. The 72^{nd} order is 4.46 times as high for the large notches compared to the case without notches, while the optimal notch shape found here reduces the 72^{nd} order to the 0.1 part.

IV. COMBINED MODEL

Pure numerical approaches for geometry optimization show very good results but are rather time-consuming. There are other approaches such as genetic algorithms varying selected geometric parameters. For a first try a fast method is favorable. Therefore, a combined numerical and analytical method is developed. The relative air-gap permeance-function, which is related to the radial component of the flux density in the air gap, derived from the FEM models is no rectangular function at all, as assumed in the analytical model before and must be derived from FE simulations. The combined model consists of two FE models using 2D magneto-static simulation and equation (4). From the first model, a machine with a slotless stator, the magnetization function of the permanent magnets is extracted. To obtain a good approximation of the relative air-gap permeance-function a second model is applied. The "permeance" model consists of the stator with slots and e.g. notches and a rotor which is magnetized uniformly in radial direction. Fig. 1 shows the results of both models for the basic model. With the two functions extracted and equation (4) the main order of the cogging torque (72^{nd}) is calculated for a variation of different notch widths. The two functions are regarded in the frequency domain. The n^{th} component of the cogging torque depends only on the n^{th} component of the square of the magnetization function and the n^{th} component of the square of the relative air-gap permeance-function. The comparison of the results of the relative cogging torque estimated for different notch widths regarded here are depicted in Fig. 2. With the assumption that the numerical optimization delivers the most accurate result the analytical and the combined method are compared to it. As described before, the analytical method does not suite the numerical results. The local minimum at $w_n = 0.17w_s$ is not detected. With the proposed combined method the cogging torque behavior is represented very good. The minimum at $w_n = 0.87w_s$ is found and there is a minimum slightly above $w_n = 0.17w_s$ at $w_n = 0.23w_s$. With the new approach a first try of optimization can be now easily be performed in a very short time. The region of interest is detected and the notch width does not have to be varied in the complete range for the numerical simulation process any longer.

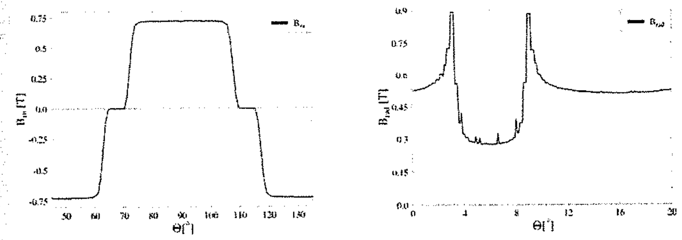

Fig. 1. Magnetization and Radial Flux-Density Function.

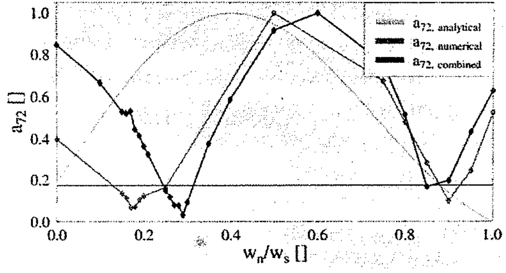

Fig. 2. Comparison of Estimated Relative Cogging Torque.

V. CONCLUSION

In this paper a new method for the shape optimization of a PMSM using a combined analytical and numerical approach is presented. The use of pure analytical methods has resulted to be not sufficiently accurate, here. Therefore, a faster optimization process that includes the use of FEM is proposed. In this process, two models are simulated: a model with a massive stator, which allows the deduction of the magnet's flux-density distribution, and a model with a radial magnetized rotor. This gives a good estimation of the relative air-gap permeance-function. The combination of the results from both models gives a good estimation of the behavior of the cogging torque. The advantage of the method is that only one rotor position per model has to be calculated, saving computational costs.

REFERENCES

[1] N. Bianchi and S. Bolognani, "Design techniques for reducing the cogging torque in surface-mounted pm motors," *IEEE Transactions on Industry Applications*, vol. 38, pp. 1259–1265, September/October 2002.

[2] S.-M. Hwang, J.-B. Eom, Y.-H. Jung, D.-W. Lee, and B.-S. Kang, "Various design techniques to reduce cogging torque by controlling energy variation in PM motors," *IEEE Transactions on Magnetics*, vol. 37, pp. 2806–2809, July 2001.