## An Effective Reluctivity Model for Non-Linear and Anisotropic Materials in Time-Harmonic Finite Element Computations

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Abstract - In time-harmonic finite element analysis, the non-linear behaviour of soft-magnetic materials is often modelled by effective reluctivity curves, to account for the time-dependence of the reluctivity during one cycle of the applied sinusoidal signal. In this paper, the effective reluctivity concept is generalised in such a way, that non-linear and anisotropic materials can be modelled as well. The model is used to simulate the flux line distribution in a three-phase transformer.

## OVERVIEW

In a time-harmonic finite element analysis,

$$\nabla \times \left(\underline{v} \nabla \times \widetilde{\widetilde{A}}\right) + j2\pi f \sigma \widetilde{\widetilde{A}} = \widetilde{\widetilde{J}}_{s}$$
 (1)

is solved, where  $\sigma$  is the electric conductivity [S/m]. The vector potential  $\tilde{\bar{A}}$  [Tm] and the current density  $\tilde{\bar{J}}_s$  [A/m²] are vectors, whose phasor-valued entries vary sinusoidally at a fixed frequency f [Hz]. The reluctivity tensor  $\underline{v}$  [m/H] generally has complex-valued entries [1]. They are set to a constant value, although this condition is not satisfied in time-domain, if non-linear materials are involved.

When the magnetic material behaviour is isotropic, it is common to obtain the (equal) entries of the reluctivity tensor from an effective reluctivity curve [1,2]. This single-valued function is often deduced from the measured magnetisation characteristic using an averaging rule. However, the flux density in a finite element evolves along an elliptic path in time and space, described by two independent parameters. If a time-harmonic analysis only considers one of them, e.g. the rms-value of the flux density  $B_{\rm rms}$  [T], additional errors are introduced. This may decrease the accuracy of the final solution in those regions where e.g. rotational fluxes are predominant. If the material additionally behaves magnetically anisotropic, an effective reluctivity model should even consider e.g. the angle  $\gamma$  between the principal axis of the flux density ellipse and the rolling direction (RD) of that material.

In Fig. 1, the average value of the reluctivity tensor entries (based on the static real-valued reluctivity model in [3]), is plotted as a function of  $\gamma$  and the aspect ratio a of the ellipses, for two different values of the maximum flux density  $B_{\text{max}}$ . In [4], a experimental method is described, which can be used to obtain equivalent models with complex-valued entries. The model in Fig. 1 is used to simulate the flux density distribution in the three-phase transformer of Fig. 2, whose core is built of Goss-textured steel. The RD is indicated by straight lines.

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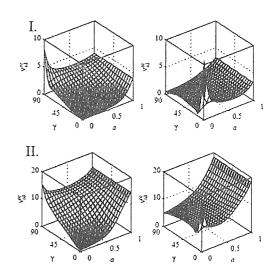


Fig. 1. Components of the reluctivity tensor  $\nu$  in the rolling direction (left) and the transverse direction (right) as a function of the aspect ratio a and the inclination angle  $\gamma$  of the B-locus relative to the rolling direction, for  $B_{\max} = 1.38$  T (top) and  $B_{\max} = 1.93$  T (bottom).

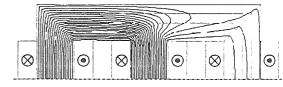


Fig. 2. Flux line distribution in a three-phase transformer. The initial phase angle in the coils equals 155° (left), 35° (middle) and -85° (right).