Determination of a dynamic radial active magnetic bearing model using the finite element method

Boštjan Polajžer, Gorazd Štumberger and Drago Dolinar
Faculty of Electrical Engineering and Computer Science, University of Maribor, Maribor, Slovenia

Kay Hameyer
Institut für Elektrische Maschinen, RWTH Aachen, Aachen, Germany

Keywords *Magnetic fields, Modelling, Nonlinear control systems*

Abstract *The dynamic model of radial active magnetic bearings, which is based on the current and position dependent partial derivatives of flux linkages and radial force characteristics, is determined using the finite element method. In this way, magnetic nonlinearities and cross-coupling effects are considered more completely than in similar dynamic models. The presented results show that magnetic nonlinearities and cross-coupling effects can change the electromotive forces considerably. These disturbing effects have been determined and can be incorporated into the real-time realization of nonlinear control in order to achieve cross-coupling compensations.*

1. Introduction

Active magnetic bearings are a system of controlled electromagnets, which enable contact-less suspension of a rotor (Schweitzer *et al.*, 1994). Two radial bearings and one axial bearing are used to control the five degrees of freedom of the rotor, while an independent driving motor is used to control the sixth degree of freedom. No friction, no lubrication, precise position control and vibration damping make active magnetic bearings (AMBs) particularly appropriate and desirable in high-speed rotating machines. Technical applications include compressors, centrifuges and precise machine tools.

The electromagnets of the discussed radial AMBs are placed on the common iron core (Štumberger *et al.*, 2000). Their behavior is, therefore, magnetically nonlinear. Moreover, the individual electromagnets are magnetically coupled. An extended dynamic AMB model is determined in this paper using the finite element method (FEM). The parameterization coupling model of the discussed radial AMBs is derived in this way. The presented dynamic AMB model is based on partial derivatives of flux linkages and radial force characteristics and, therefore, describes magnetic nonlinearities and cross-coupling effects more completely than similar dynamic AMB models (Antila *et al.*, 1998; Štumberger *et al.*, 2000). Moreover, it is appropriate for nonlinear control design and is compact and fast enough for the real-time realization.

FEM-computed force is compared with the measured force, while the flux linkages were not measured due to mechanical problems with rotor fixation. The current and position dependent partial derivatives of flux linkages are calculated by analytical derivations of the continuous approximation functions of the FEM-computed flux linkages. The impact of magnetic nonlinearities and cross-coupling effects on

COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering
Vol. 23 No. 3, 2004
pp. 783-788

0332-1649
DOI 10.1108/03321640410540728

the properties of the discussed radial AMBs is then evaluated based on the performed calculations.

2. Dynamic AMB model

The dynamic AMB model is according to the circuit model presented in Figure 1 given by equations (1) and (2), where u_1, u_2, u_3 and u_4 are the supply voltages, I_0 is the constant bias current, $i_{x\Delta}$ and $i_{y\Delta}$ are the control currents in the x- and y-axis. ψ_1, ψ_2, ψ_3 and ψ_4 are the flux linkages of the corresponding electromagnets. R stands for the coil resistances. F_x and F_y are the radial force components in the x- and y-axis, m is the mass of the rotor.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = R \begin{bmatrix} I_0 + i_{x\Delta} \\ I_0 - i_{x\Delta} \\ I_0 + i_{y\Delta} \\ I_0 - i_{y\Delta} \end{bmatrix} + 2 \begin{bmatrix} \frac{\partial \psi_1}{\partial i_{x\Delta}} & \frac{\partial \psi_1}{\partial i_{y\Delta}} \\ \frac{\partial \psi_2}{\partial i_{x\Delta}} & \frac{\partial \psi_2}{\partial i_{y\Delta}} \\ \frac{\partial \psi_3}{\partial i_{x\Delta}} & \frac{\partial \psi_3}{\partial i_{y\Delta}} \\ \frac{\partial \psi_4}{\partial i_{x\Delta}} & \frac{\partial \psi_4}{\partial i_{y\Delta}} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d} i_{x\Delta}}{\mathrm{d}t} \\ \frac{\mathrm{d} i_{y\Delta}}{\mathrm{d}t} \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_1}{\partial x} & \frac{\partial \psi_1}{\partial y} \\ \frac{\partial \psi_2}{\partial x} & \frac{\partial \psi_2}{\partial y} \\ \frac{\partial \psi_3}{\partial x} & \frac{\partial \psi_3}{\partial y} \\ \frac{\partial \psi_4}{\partial x} & \frac{\partial \psi_4}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} \\ \frac{\mathrm{d}y}{\mathrm{d}t} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \\ \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} F_x(i_{x\Delta}, x) \\ F_y(i_{y\Delta}, y) \end{bmatrix} \tag{2}$$

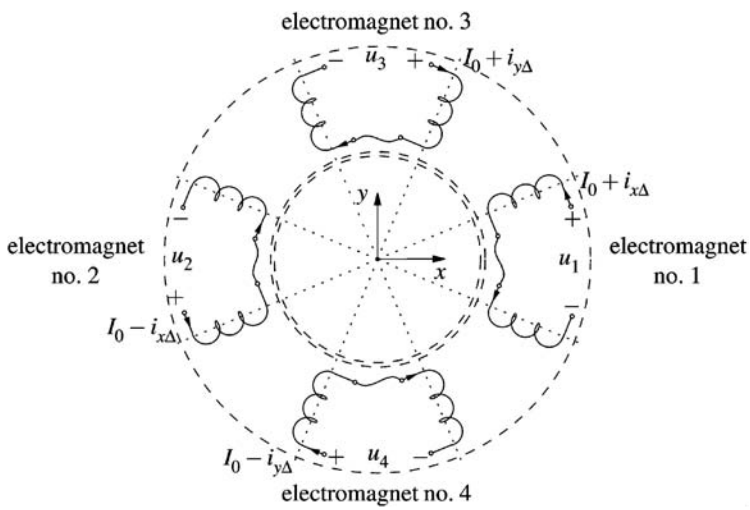

Figure 1.
The circuit AMB model

The current and position dependent partial derivatives of the flux linkages required in equations (1) are calculated by analytical derivations of the continuous approximation functions of FEM-computed flux linkages. The force characteristics $F_x(i_{x\Delta},x)$ and $F_x(i_{y\Delta},y)$ required in equations (2) are determined by FEM. In this way, the obtained dynamic AMB model (1), (2) is described in terms of parameterization coupling.

When considering the symmetry in geometry (Figure 2), and the differential driving mode of currents $i_1 = I_0 + i_{x\Delta},\ i_2 = I_0 - i_{x\Delta},\ i_3 = I_0 + i_{y\Delta}$ and $i_4 = I_0 - i_{y\Delta}$, the interaction between electromagnets in the x-axis (no. 1 and no. 2) and electromagnets in the y-axis (no. 3 and no. 4) can be expressed as equations (3) and (4).

$$\frac{\partial \psi_1}{\partial i_{x\Delta}} = \frac{\partial \psi_3}{\partial i_{y\Delta}},\quad \frac{\partial \psi_1}{\partial i_{y\Delta}} = \frac{\partial \psi_3}{\partial i_{x\Delta}},\quad \frac{\partial \psi_2}{\partial i_{x\Delta}} = \frac{\partial \psi_4}{\partial i_{y\Delta}},\quad \frac{\partial \psi_2}{\partial i_{y\Delta}} = \frac{\partial \psi_4}{\partial i_{x\Delta}} \tag{3}$$

$$\frac{\partial \psi_1}{\partial x} = -\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial y} = -\frac{\partial \psi_4}{\partial y},\quad \frac{\partial \psi_1}{\partial y} = -\frac{\partial \psi_2}{\partial y} = \frac{\partial \psi_3}{\partial x} = -\frac{\partial \psi_4}{\partial x} \tag{4}$$

The electromotive forces (EMFs) due to the magnetic nonlinearities are reflected in terms like $\partial \psi_3/\partial i_{y\Delta}$ and $\partial \psi_3/\partial y$, which are normally given as constant inductance and speed coefficient, respectively (Schweitzer *et al.*, 1994). In the work of Antila *et al.* (1998) magnetic nonlinearities are partially considered with dynamic inductance. However, the EMFs due to cross-coupling effects, which are reflected in terms like $\partial \psi_1/\partial i_{y\Delta}$ and $\partial \psi_1/\partial y$, are neglected by Antila *et al.* (1998). The dynamic AMB model (1), (2), therefore, describes magnetic nonlinearities and cross-coupling effects more completely than similar dynamic models. Furthermore, it is appropriate for nonlinear control design and is compact and fast enough for the real-time realization.

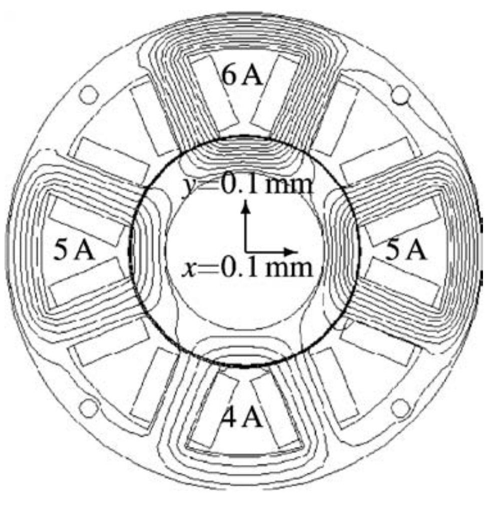

Figure 2.
The geometry and field distribution of the discussed radial AMBs

3. FEM computation of flux linkage and radial force characteristics

Magneto-static computation was performed by 2D FEM. The geometry and magnetic field distribution of the discussed radial AMBs is shown in Figure 2. The flux linkage characteristics $\psi_1(i_{x\Delta}, i_{y\Delta}, x, y)$, $\psi_2(i_{x\Delta}, i_{y\Delta}, x, y)$, $\psi_3(i_{x\Delta}, i_{y\Delta}, x, y)$ and $\psi_4(i_{x\Delta}, i_{y\Delta}, x, y)$ were calculated in the entire operating range from the average values of the magnetic vector potential in the stator coils. The radial force characteristics $F_x(i_{x\Delta}, x)$ and $F_y(i_{y\Delta}, y)$ were also calculated in the entire operating range by the Maxwell's stress tensor method, where integration was performed over a contour placed along the middle layer of the three-layer air gap mesh. The obtained results were incorporated into the extended dynamic AMB model (1) and (2). The parameterization coupling model is derived in this way.

4. Results

The magnetic properties of the rotor surface changed due to the manufacturing process of the rotor steel sheets. Therefore, the magnetic air gap became larger than the geometric one. In order to obtain good agreement between the calculated and measured forces in the linear region, the air gap was increased in the FEM computation from 0.4 to 0.45 mm. The increase in the air gap of 0.05 mm can be compared with the findings by Antila *et al.* (1998).

A good agreement between the FEM-computed and the measured radial force characteristics can be seen in Figure 3(a) and (b). The current and position dependent partial derivatives of flux linkages, shown in Figure 3(c)-(f), were calculated by analytical derivations of the continuous approximation functions. In the results shown in Figure 3(c) and (d) the influence of magnetic nonlinearities can be seen, while the influence of magnetic cross-coupling effects can be seen in the results shown in Figure 3(e) and (f). Based on the obtained results the ratio $(\partial\psi_1/\partial i_{y\Delta})/(\partial\psi_3/\partial i_{y\Delta})$, as well as the ratio $(\partial\psi_1/\partial y)/(\partial\psi_3/\partial y)$ was calculated inside the operating range. From the performed comparison, it is established that due to magnetic nonlinearities and cross-coupling effects the EMFs can vary in a range of up to 12 percent.

5. Conclusion

The extended dynamic AMB model is presented in this paper. It is based on the FEM-computed current and position dependent partial derivatives of flux linkages and radial force characteristics. The parameterization coupling model of the discussed radial AMBs is derived in this way. The obtained dynamic AMB model, therefore, considers magnetic nonlinearities and cross-coupling effects more completely than similar dynamic AMB models. The results of the performed calculations show that inside the operating range of the discussed radial AMBs, the EMFs can vary due to magnetic nonlinearities and cross-coupling effects in a range of up to 12 percent. These disturbing effects deteriorate the static and dynamic performances of the overall system. In order to improve the system dynamics, the obtained results have to be incorporated into the real-time realization of nonlinear control.

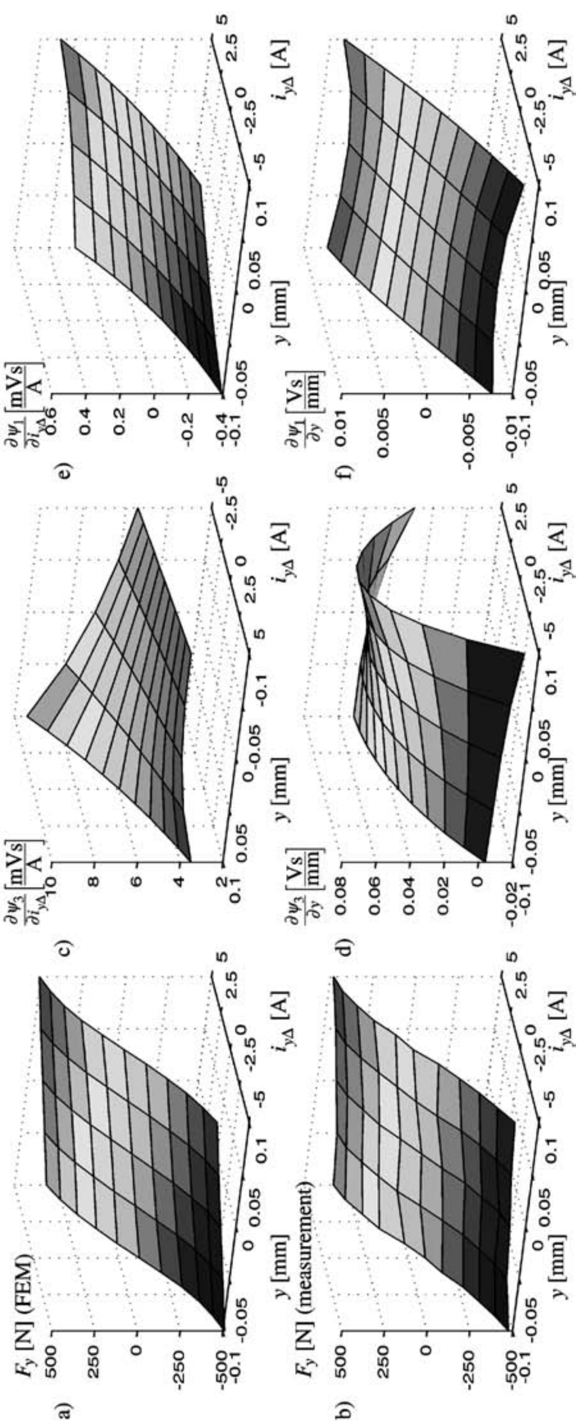

Figure 3.
Results for the case: $x = 0\,\mathrm{mm}$, $i_{x\Delta} = 0\,\mathrm{A}$ and $I_0 = 5\,\mathrm{A}$: calculated and measured force (a and b) and flux linkage partial derivatives (c, d, e and f)

References

Antila, M., Lantto, E. and Arkkio, A. (1998), "Determination of forces and linearized parameters of radial active magnetic bearings by finite element technique", *IEEE Trans. Magn.*, Vol. 34 No. 3, pp. 684-94.

Schweitzer, G., Bleuler, H. and Traxler, A. (1994), *Active Magnetic Bearings*, Vdf Hochschulverlag AG an der ETH Zürich, Zürich.

Štumberger, G., Dolinar, D., Pahner, U. and Hameyer, K. (2000), "Optimization of radial active magnetic bearings using the finite element technique and the differential evolution algorithm", *IEEE Trans. Magn.*, Vol. 36 No. 4, pp. 1009-13.