CONVERGENCE BEHAVIOUR OF DIFFERENT FORMULATIONS FOR TIME-HARMONIC AND TRANSIENT EDDY-CURRENT COMPUTATIONS IN 3D

D van Riesen, C Kaehler, G Henneberger

Aachen University (RWTH), Germany

Eddy-current calculation in electromagnetical devices is becoming more important, as both the available computational power and the need for power loss reduction increase. Several different formulations for the calculation of eddy currents using the finite-element method (FEM) have been proposed. For 3D applications in static cases the magnetic vector potential ($\vec{A}$) with edge elements is the most common case. The static formulation can be extended to account for eddy current effects using an extra term with the same magnetic vector potential, with an electric vector potential ($\vec{T}$) or with an electric scalar potential (V). The $\vec{A}-\vec{A},V$ formulation is commonly believed to be the best in terms of stability and convergence rate. In this paper the three different formulations, in a time-harmonic and a transient case for a simple C-core model, the problem given in the TEAM Workshop No. 7, and for a claw-pole alternator are compared. The TEAM Workshop problem is also verified against measurement results as obtained by Fujiwara et al (1). All solvers are part of the iMOOSE software package (2).

FORMULATIONS

A very thorough overview over the above mentioned formulations for eddy-current analysis is given by Bíró (3) for the time-harmonic analysis. For the transient calculation, the formulations are extended with the time-dependent values discretized according to the Galerkin scheme, resulting e.g. for the magnetic vector potential:

$$\vec{A}(t) = (1-\Theta)\vec{A}_n + \Theta A_{n+1} \quad (1)$$

$$\Theta = \frac{t-t_n}{t_{n+1}-t_n} = \frac{t-t_n}{\delta t}; \quad 0 \leq \Theta \leq 1 \quad (2)$$

with $\Theta = \frac{2}{3}$, cf. Zienkiewicz et al (4).
All three formulations share a common formulation for non-eddy-current regions which is equal to the static case (with the added time dependency) and is therefore not displayed here. The $\vec{A}-\vec{A},V$ formulation in Galerkin form reads in eddy-current regions:

$$\int_{\Omega_n,\Omega_c} \tau \nabla\times \vec{N}_i \cdot \nu \nabla\times \vec{A}_{n+1}\, \mathrm{d}\,\Omega + \int_{\Omega_c} \frac{\sigma}{\Delta t}\vec{N}_i \cdot \vec{A}_{n+1}\, \mathrm{d}\,\Omega + \int_{\Omega_c} \frac{\sigma}{\Delta t}\vec{N}_i \cdot \nabla\cdot V_{n+1}\, \mathrm{d}\,\Omega =$$
$$\int_{\Omega_c} (\tau-1)\nabla\times \vec{N}_i \cdot \nu \nabla\times \vec{A}_n\, \mathrm{d}\,\Omega + \int_{\Omega_c} \frac{\sigma}{\Delta t}\vec{N}_i \cdot \vec{A}_n + \int_{\Omega_c} \frac{\sigma}{\Delta t}\vec{N}_i \cdot \nabla\cdot V_n\, \mathrm{d}\,\Omega +$$
$$\int_{\Omega_c} \nabla\times \vec{N}_i \cdot \left(\tau\vec{T}_{0,n+1} + (1-\tau)\vec{T}_{0,n}\right) \mathrm{d}\,\Omega \quad \forall \vec{N}_i \quad (3)$$

$$\int_{\Omega_c} \frac{\sigma}{\Delta t}\nabla\cdot N_i \cdot \vec{A}_{n+1}\, \mathrm{d}\,\Omega + \int_{\Omega_c} \frac{\sigma}{\Delta t}\nabla\cdot N_i \cdot \nabla\cdot V_{n+1}\, \mathrm{d}\,\Omega =$$
$$\int_{\Omega_c} \frac{\sigma}{\Delta t}\nabla\cdot N_i \cdot \vec{A}_n\, \mathrm{d}\,\Omega + \int_{\Omega_c} \frac{\sigma}{\Delta t}\nabla\cdot N_i \cdot \nabla\cdot V_n\, \mathrm{d}\,\Omega \quad \forall N_i \quad (4)$$

The $\vec{A}-\vec{A},\vec{T}$ formulation is given here in Galerkin form for eddy-current regions:

$$\int_\Omega \left[\tau\nabla\times\ \vec{N}_i\ \nu\ \nabla\times\vec{A}_{n+1} - \tau\vec{N}_i\ \nabla\times\vec{T}_{n+1}\right] d\Omega =$$
$$\int_\Omega \Big[(\tau-1)\nabla\times\vec{N}_i\cdot\nu\ \nabla\times\ \vec{A}_n - (\tau-1)\vec{N}_i\ \nabla\times\vec{T}_n + \tau\vec{N}_i\vec{J}_{0n+1} + (\tau-1)\vec{J}_{0n}$$
$$+ \nabla\times\ \vec{N}_i\ \nu\big(\tau\vec{B}_{rn+1} + (1-\tau)\vec{B}_{rn}\big)\Big] d\Omega \quad \forall\vec{N}_i \quad (5)$$

$$\wedge \quad \int_\Omega \left(\tau\ \nabla\times\ \vec{N}_i\cdot\frac{1}{\sigma}\ \nabla\times\ \vec{T}_{n+1} + \nabla\times\ \vec{N}_i\cdot\frac{1}{\Delta t}\vec{A}_{n+1}\right) d\Omega$$
$$= \int_\Omega \left((\tau-1)\ \nabla\times\ \vec{N}_i\cdot\frac{1}{\sigma}\ \nabla\times\ \vec{T}_n + \nabla\times\ \vec{N}_i\cdot\frac{1}{\Delta t}\vec{A}_n\right) d\Omega \quad \forall\vec{N}_i \quad (6)$$

The $\vec{A},\vec{A}^*$ formulation is not given here as it can be deducted from (3) by suppressing the term with the electric scalar potential.

RESULTS

Three different models are used to test the convergence behaviour of the different formulations. A simple C-Core with a conducting plate in the yoke, the TEAM Workshop Problem No 7 (with results that are verified against the measurements by Fujiwara et al (1)) and a claw pole alternator as a larger scale problem. The $\vec{A}-\vec{A},\vec{T}$ formulation as implemented in iMOOSE is not suited for the multiply connected eddy-current region of the TEAM problem, and does not converge for realistic conductivity values in the case of the claw pole alternator. A comparison between the $\vec{A}$ and $\vec{A}-\vec{A},\vec{T}$ formulations was done by the authors in (5).

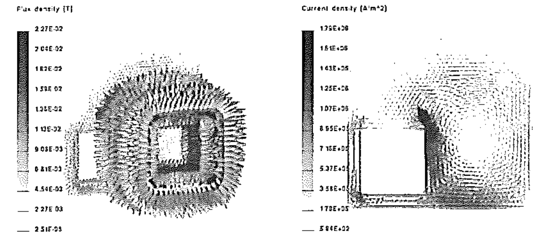


Figure 1: Flux-density and current-density distribution for TEAM Workshop No 7

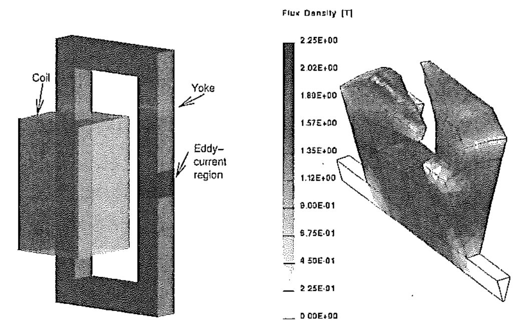
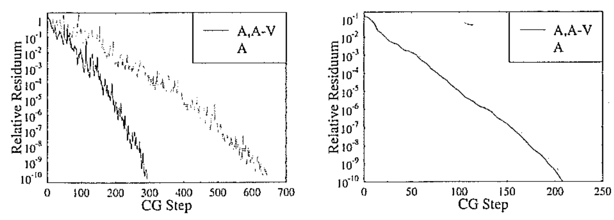


Figure 2: Model of the yoke and the flux density distribution on the claws of the claw pole alternator

Figs. 3 and 4 and tables 1 and 2 show the convergence of different calculations. For the time-harmonic analysis, both for the C-Core and the TEAM Workshop, the convergence behaviour is significantly better for the $\vec{A}-\vec{A},V$ -formulation, as expected. For the claw pole alternator, the $\vec{A}-\vec{A},V$ - formulation requires slightly less CG iterations, at the cost of more Newton iterations (8 vs. 6) and more unknowns as compared to the $\vec{A}$ -formulation. Thus, the execution times are similar, even a little longer for the $\vec{A}$ -formulation. Nevertheless, as stated in (5), the $\vec{A}$ and $\vec{A}-\vec{A},\vec{T}$ -formulations have stability problems at lower and higher conductivities respectively, while the $\vec{A}-\vec{A},V$ -formulation is stable over the whole range of frequencies and conductivities.

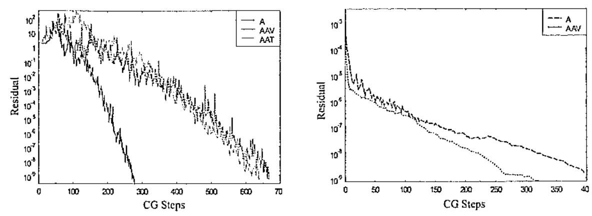
Table 1: Time, number of CG steps, and matrix dimension for the time-harmonic case, TEAM Problem

Formulation	time (sec.)	# steps	Dim.
$\vec{A}$	1258	645	213200
$\vec{A}-\vec{A},V$	800	298	219864



(a) Harmonic (b) Transient

Figure 3: Convergence for the harmonic and transient calculation of TEAM Workshop Problem No 7



(a) C-Core, harmonic (b) Clawpole, transient

Figure 4: Convergence for the harmonic calculation of the C-Core model and the transient calculation of the claw pole alternator

Table 2: Time, number of CG steps, and matrix dimension for the transient case, TEAM problem

Formulation	time (sec.)	# steps	Dim.
$\vec{A}$	44882	135	213200
$\vec{A}-\vec{A},V$	48387	140	219864

CONCLUSION

The $\vec{A}-\vec{A},V$ -formulation is generally believed to give faster and more stable convergence for eddy-current problems than the $\vec{A}$ or the $\vec{A}-\vec{A},\vec{T}$ -formulation. This can be reproduced in the time-harmonic case, while in the transient case there is not such a huge advantage. This has to be further investigated.

1. Fujiwara, K. and Nakata, T., April 1990, Compel, 9, 3, 137–154
2. Innovative Modern Object-Oriented Solving Environment - iMOOSE, Available: http://www.imoose.de, [Online]
3. Bíró, O., 1999, Comput. Methods Appl. Mech. Engrg., 169, 391–405
4. Zienkiewicz, O. and Taylor, R., 1989, The finite element method, vol. 1, McGraw-Hill Book Company, London
5. Kaehler, C., van Riesen, D., Albertz, D., and Henneberger, G., 2002, in Proceedings of the IGTE'02, 88, Graz, Austria