Comparison of 2D and 3D Transient FEM Calculations of a Skewed Induction Machine

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***Abstract*— The Finite-Element Method (FEM) allows for the calculation of the magnetic field distribution and the torque in electrical machines. Transient vector formulations are utilized to compute the torque of a skewed induction machine. In the two-dimensional approach three slices are calculated to take the skew of the rotor into account. Both approaches, the two-dimensional (2D) and the three-dimensional (3D), consider the movement of the rotor geometry of the machine and calculate the currents in the rotor bars. The 2D calculation is preferred because of its time efficiency. The aim of the presented paper is to examine, wether the two-dimensional calculation provides sufficiently accurate results in comparison to the expensive 3D computation.**

***Index Terms*—Transient FEM Computation, Skewed Induction Machine, Induced Currents, Rotor Movement, Comparison 2D - 3D**

I. Introduction

Induction machines are used for all kinds of actuation in industry and are therefore also matter of research. A main problem of all electrical machines is the noise evaluation. To predict the noise radiation of an electrical drive, the magnetic force and the torque of the machine, both exiting the vibrations, have to be calculated exactly.

Due to higher harmonics in the noise evaluation a transient calculation has to be conducted with a high time resolution. To take the skew of the rotor into account, a 3D approach has to be employed. 3D FEM calculations are very expensive concerning calculation time and memory effort. Therefore, a sufficiently exact 2D FEM approach has to be preferred.

Corresponding to the 2D approach it is not possible to take the skew of the machine directly into account. Hence, three slices of the machine are calculated. For the slice positioned in the middle of the machine, a transient computation is applied in which the currents in the rotor bars are determined. With these currents, two static FEM calculations are performed on the two remaining slices. In comparison with the middle slice of the machine, the rotor in the other slices is turned according to the skew of the machine.

For the interpretation of the results, the torque of the three-dimensional and the two-dimensional calculations are compared. To evaluate the torque of the 2D calculations the results of the three slices are superposed.

In order to calculate the machine in 2D and 3D numerical tools for the design process have been developed. In this paper the 2D and the 3D approaches are outlined, the FEM models are described and the results of the calculations are compared.

II. Theory of the Transient Solvers

The applied solvers are parts of an object-oriented solver package [1]. They apply a transient FEM formulation taking the rotational movement into account.

The node-based 2D $\vec{A}$-approach applies the magnetic vector potential in all regions. The following equation (already in Galerkin formulation) is solved in all regions Γ [2]:

$$\int_\Gamma \left(\nabla\cdot \alpha_i \cdot \nu \cdot \nabla\cdot A_z(t) + \alpha_i \cdot \sigma \frac{\partial}{\partial t} A_z(t)\right) \mathrm{d}\Gamma = \int_\Gamma (\alpha_i \cdot J_{z0}(t))\,\mathrm{d}\Gamma \quad . \tag{1}$$

The material parameters ν and σ represent the non-linear reluctivity and the linear conductivity, respectively. α_i defines the shape function of an element (in this solver triangles). $J_{z0}(t)$ describes the z-component of the given coil current density $\vec{J_0}(t)$. Note, that $\vec{J_0}(t) > \vec{0}$ only applies for $\sigma = 0$.

The edge-based 3D $\vec{A}$-approach applies also the magnetic vector potential $\vec{A}$ in all regions Ω. The following equation (again in Galerkin formulation) is solved [3]:

$$\int_\Omega \left(\nabla\times \vec{\alpha}_i \cdot \nu \nabla\times \vec{A}(t) + \alpha_i \cdot \sigma \frac{\partial}{\partial t}\vec{A}(t)\right) d\Omega = \int_\Omega \left(\vec{\alpha}_i \cdot \vec{J_0}(t)\right) d\Omega \quad . \tag{2}$$

$\vec{\alpha}_i$ defines the shape function of a 3D edge element (in this solver tetrahedra).

The time-stepping algorithm interpolates the time-dependent variables linearly:

$$\vec{A}(t) = \tau \cdot \vec{A}_{n+1} + (1-\tau)\vec{A}_n \qquad \text{ditto for } \vec{J_0};$$
$$\frac{\partial}{\partial t}\vec{A}(t) = \frac{1}{\Delta t}(\vec{A}_{n+1} - \vec{A}_n) \quad , \tag{3}$$

where n represents the number of the transient step, Δt the time in between transient steps and τ the relaxation factor. The relaxation factor used in between transient steps is chosen as $\tau = \frac{2}{3}$ (Galerkin scheme) [4].

The magnetic flux density $\vec{B}$ and the eddy-current density $\vec{J}$ are computed from the magnetic vector potential as follows:

$$\vec{B} = \nabla\times \vec{A}, \qquad \vec{J} = -\sigma \frac{A_{n+1} - A_n}{\Delta t} \quad . \tag{4}$$

The resulting global matrix of both approaches is symmetric, thus allowing the storage as lower or upper triangular matrix and the use of the Cholesky-CG combination [5] of the ITL package [6]. Saturation effects are computed with an overlaying Newton-Raphson procedure for each transient step.

In the 2D approach, the rotor mesh of the machine is turned and the air-gap is remeshed for each transient step. The benefit of the remeshing strategy lies in a time stepping independent of the mesh of the machine and thus an arbitrary rotation angle.

To represent the rotational movement in the 3D model a lock-step method is utilized. In this method the movement is purely virtual. Boundary conditions are used which pair edges in a sliding area mesh. In each step a search function connects the edges in this area depending on the displacement in between the transient steps, while the mesh remains stationary [7]. Thus, the flux is coupled between moving and stationary regions.

III. Finite-Element Models

Only magnetically relevant components of the induction machine are modelled. Since the geometry of the machine is symmetric, antiperiodic boundaries reduce the FEM model to one pole pitch or 90° .

The 3D model of the induction machine is depicted in Fig. 1 on the right with translucent iron regions. With this model the torque of the machine is determined and compared to the results of the 2D calculations on the model in Fig. 1 on the left.

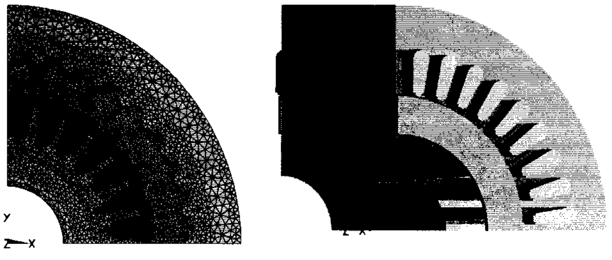

Fig. 1
2D and 3D models of the induction machine

IV. Calculations and Results

All calculations are conducted at a constant speed of $n = 970$ rpm. The mechanical step angle amounts to $\alpha = 4°$, leading to $\Delta t = 0.68728$ ms in between transient steps. The three-phase current of $I = 472$ A is impressed into the stator coils. The material conductivity of aluminum $\sigma = 1.834 \cdot 10^7 [\Omega m]^{-1}$ at material temperature $T = 200°$C is used for the massive aluminum regions of the rotor cage.

The rotor-bar currents are automatically induced by the electro-magnetic field in the 2D calculation of the middle slice and in the 3D calculation. In the other slices the computed rotor-bar currents are applied. The end ring of the rotor cage is taken into consideration as well.

The 2D calculated torque in time and frequency domain is depicted in Fig. 2 (time domain for middle slice). The resulting torque is generated by superposition of the torque values of all slices (frequency domain for superposition). The results of the 3D computation are shown in Fig. 3.

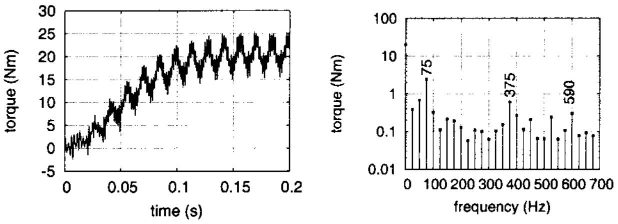

Fig. 2
Torque of the 2D calculation

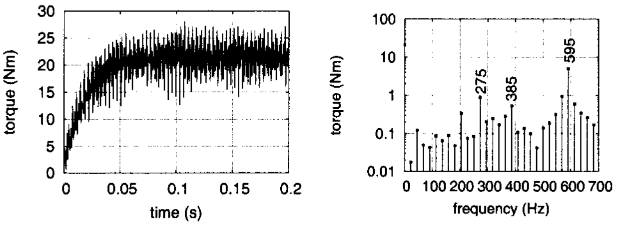

Fig. 3
Torque of the 3D calculation

The frequency spectra of the 2D and the 3D calculations correspond rather good in the stationary value of the torque. Additionally both calculations show leading amplitudes at frequencies of approximately $f = 390$ Hz and $f = 600$ Hz.

V. Conclusion

In this paper a two-dimensional and a three-dimensional approach for the calculation of a skewed induction machine are compared. In the full paper 2D calculations with more than three slices of the machine will be presented and compared with the three-dimensional calculation. Additionally, the differences in the frequency domain will be investigated.

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