On the Use of the New Edge Based $\vec{A} - \vec{A}, \vec{T}$ Formulation for the Calculation of Time-Harmonic, Stationary and Transient Eddy Current Field Problems

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***Abstract*—Most papers concerning the calculation of 3D eddy current problems are using a combination of a vector potential and a scalar potential to solve the electromagnetic field in the eddy current regions. This paper uses the $\vec{A} - \vec{A}, \vec{T}$ formulation with both the magnetic vector potential $\vec{A}$ and the electric vector potential $\vec{T}$ in the eddy current regions. Since nodal vector potentials with continuous normal components have accuracy problems at interfaces of regions with different permeabilities, edge elements can be used for both potentials. The formulation is applied to the calculation of stationary eddy current field problems induced by motion, time-harmonic and transient eddy current field problems.**

***Index Terms*—Convergence of numerical methods, eddy currents, electromagnetic fields, finite element methods.**

I. Introduction

FOR THE calculation of 3D eddy current fields mostly two different potential formulations have been used. The $\vec{A}, V$ formulation needs the magnetic vector potential $\vec{A}$ and the electric scalar potential V, the $\vec{t}, \Phi$ formulation the electric vector potential $\vec{T}$ and the magnetic scalar potential Φ to compute the eddy current distribution in the conducting regions [1]–[4]. Because of the lack of accuracy of nodal elements edge elements are used for the vector potentials [5], [6]. However, combining an edge based vector potential with a scalar nodal based potential in the $\vec{A}, V$ formulation or in the $\vec{t}, \Phi$ formulation can lead to a collapsing convergence process [7], [8]. Using the new $\vec{A}-\vec{A}, \vec{T}$ formulation presented in [8], which employs two edge based vector potentials in the eddy current regions, the mentioned problems of the $\vec{A}, V$ formulation or $\vec{t}, \Phi$ formulation can be eliminated [8]. This paper extends the $\vec{A} - \vec{A}, \vec{T}$ formulation to different types of eddy current field problems.

II. The Element Type of the $\vec{A}, \vec{T}$-Formulation

The presented $\vec{A}-\vec{A}, \vec{T}$ formulation for the different types of eddy current field problems can be realized by the use of edge elements as well as by the use of nodal elements. But, as already mentioned, it is the advantage of the presented formulation that all potentials can be approximated by the more accurate edge based vector potentials. In Fig. 1 the degrees of freedom for $\vec{A}$ and $\vec{T}$ in a tetrahedral edge element are displayed.

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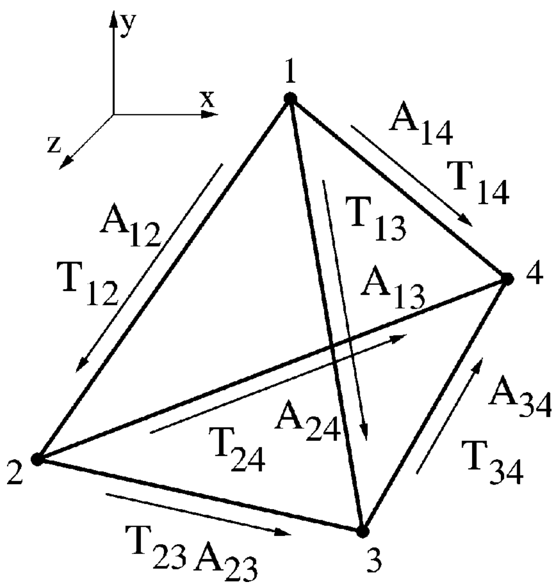

Fig. 1. Interpolation function in the edge based $\vec{A}, \vec{T}$ formulation.

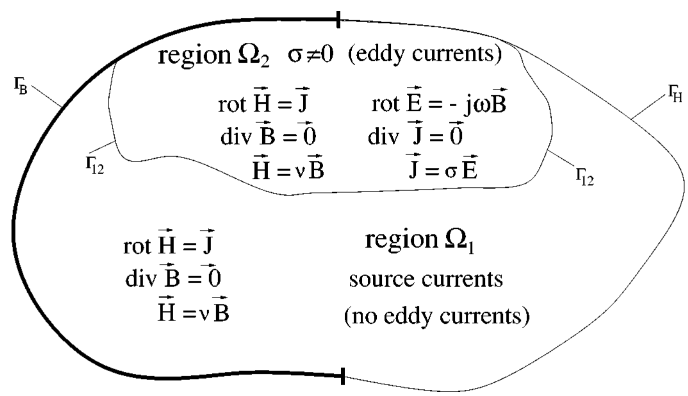

Fig. 2. Time-harmonic eddy current field problem.

III. Time-Harmonic Eddy Current Problems

A. Formulation

Fig. 2 shows the different regions in a time-harmonic eddy current field problem with the corresponding Maxwell equations. The excitation regions, in Fig. 2 region 1, can be considered by the $\vec{A}$ formulation solving

$$\int_\Omega \text{curl}\, \vec{\alpha}_i \cdot \nu \text{curl}\, \vec{A}\, d\Omega = \int_\Omega \vec{\alpha}_i \cdot \vec{J}\, d\Omega.$$

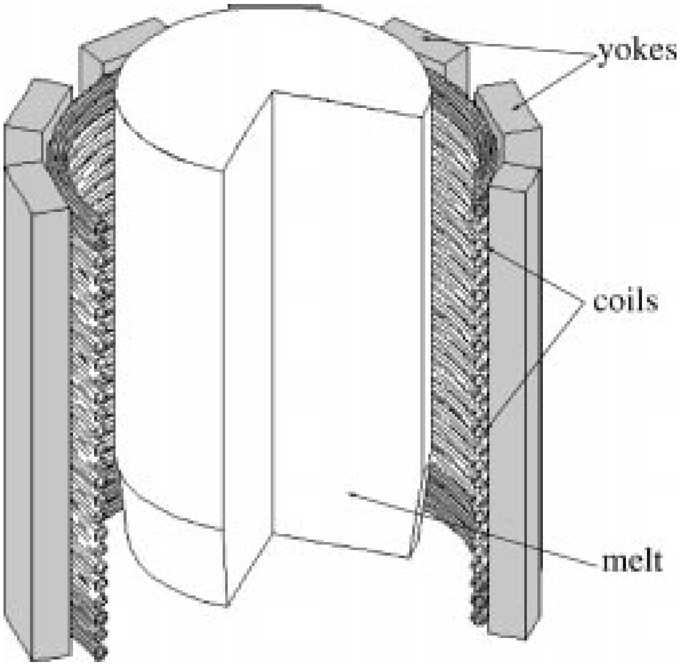

Fig. 3. Structure of an induction furnace.

In the eddy current region Ω_2 the new edge based $\vec{A}, \vec{T}$ formulation is used to solve both the magnetic and electric field distribution. The equations to solve the edge based degrees of freedom for $\vec{A}$ and $\vec{T}$ are given in [8]:

$$\int_\Omega (\text{curl } \vec{\alpha}_i \cdot \nu \text{curl } \vec{A} - \vec{\alpha}_i \cdot \text{curl } \vec{T})\, d\Omega = 0$$
$$\int_\Omega (\text{curl } \vec{\alpha}_i \cdot \frac{1}{\sigma}\text{curl } \vec{T} + \text{curl } \vec{\alpha}_i \cdot j\omega\vec{A})\, d\Omega = 0$$

B. Application

The presented formulation is applied to the calculation of the electromagnetic eddy current field of an induction furnace shown in Fig. 3. A water cooled copper coil causes a time harmonic field of 500 Hz which lead to high eddy currents heating up the melt. For optimizing the power efficiency of the induction furnace the current density distribution in the coil turns is of high interest.

Fig. 4 pictures the 3D eddy current distribution in the upper coil turns of the furnace. By knowing exactly the inhomogeneous current density distribution in the coils the losses of the coil can be determined and the efficiency of the furnace can be determined by that method. Because of the high permeability difference at the yoke-air or the melt-air interface a calculation with nodal vector element solvers would lead to inaccuracies in the solution. Using the presented pure edge based formulation these inaccuracies are avoided.

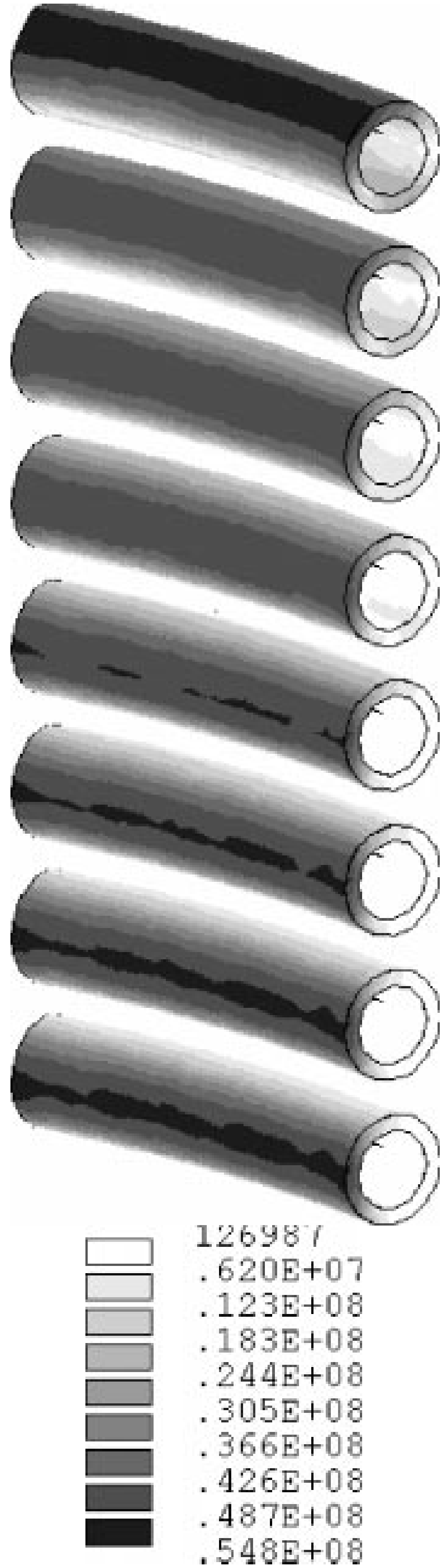

Fig. 4. Absolute value of the real part of the current density $\vec{J}$ in A/m^2.

IV. Eddy Current Problems Induced by Motion

A. Formulation

Eddy currents can also be induced by a movement of a conductor through a magnetic field. If the geometry of the model stays invariant during the movement, the movement can be considered by the velocity term $\vec{v} \times \vec{B}$ in the coordinate system of

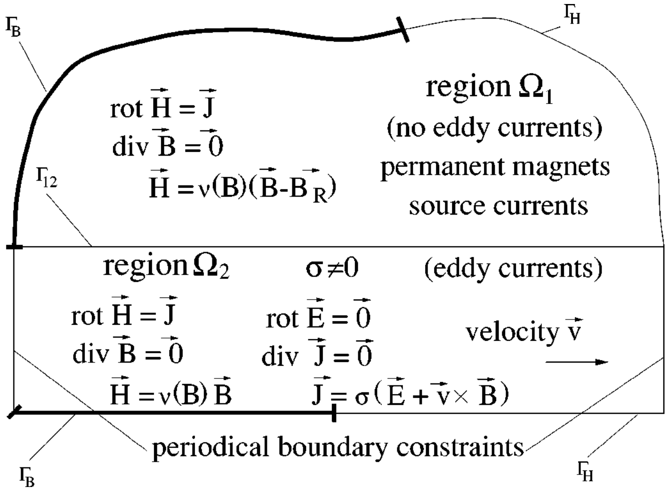

Fig. 5. Eddy current field problem induced by motion.

the excitation regions. Fig. 5 shows a configuration for an eddy current field problem induced by motion. In case of a magnetostatic excitation nonlinear magnetization characteristics can be considered in both regions. Using the $\vec{A}$ formulation in the region Ω_1 results in the following system equation:

$$\int_\Omega \text{curl}\, \vec{\alpha}_i \cdot \nu \text{curl}\, \vec{A}\, d\Omega = \int_\Omega (\vec{\alpha}_i \cdot \vec{J} + \text{curl}\, \vec{\alpha}_i \cdot \vec{B}_r)\, d\Omega.$$

The $\vec{A}$, $\vec{T}$ formulation in region Ω_2 leads to the equations

$$\int_\Omega (\text{curl}\, \vec{\alpha}_i \cdot \nu \text{curl}\, \vec{A} - \vec{\alpha}_i \cdot \text{curl}\, \vec{T})\, d\Omega = 0$$

$$\int_\Omega \left(\text{curl}\, \vec{\alpha}_i \cdot \frac{1}{\sigma}\, \text{curl}\, \vec{T} - \text{curl}\, \vec{\alpha}_i \cdot \vec{v} \times \text{curl}\, \vec{A}\right) d\Omega =$$

The nonlinearity of the iron parts is considered by the Newton–Raphson method. Because of convergence problems caused by the velocity term in case of high speeds, it is only possible to solve problems with low velocities. Upwinding techniques applied to this formulation do not have the same improvement of the convergence process as it is known from nodal vector potential solvers.

B. Application

Fig. 6 shows one of the four hybrid carrying magnets of a magnetic levitation vehicle that operates with velocities up to 5 km/h. Because of the movement of the magnet relative to the fixed railway eddy currents are induced in the railway which leads to different retarding and normal forces dependent on the speed. For the levitation control it is important to know the different force behavior dependent on the speed. Using the presented formulation it is possible to consider the movement of the magnets by the moving term $\int_\Omega \text{curl}\, \vec{\alpha}_i \cdot \vec{v} \times \text{curl}\, \vec{A}\, d\Omega$.

The resulting eddy currents in the nonlinear iron rail are displayed in Fig. 7. Considering the eddy currents in the rail it is possible to compute correctly the normal and retarding forces of the levitation system to get the parameter for the levitation control.

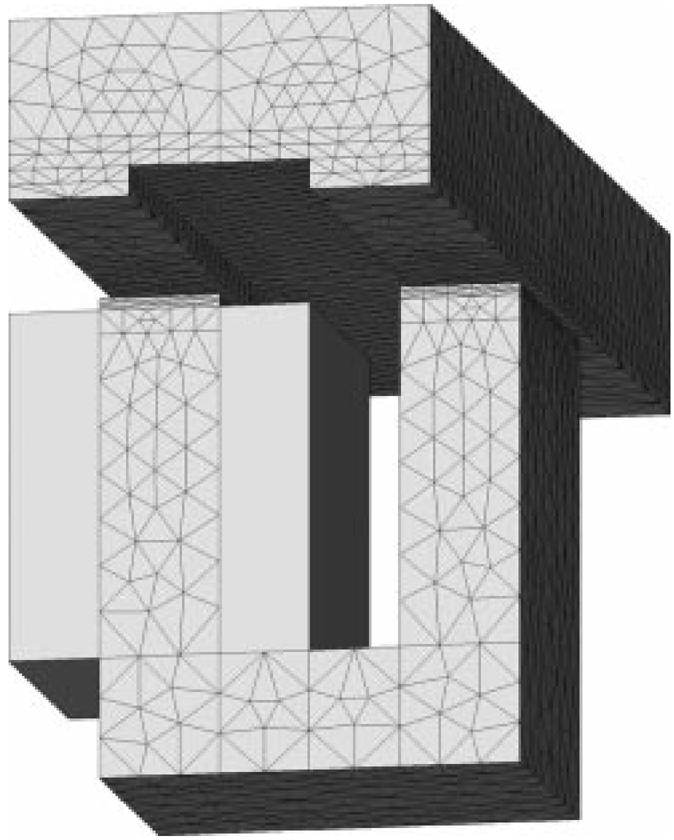

Fig. 6. Hybrid levitation magnet.

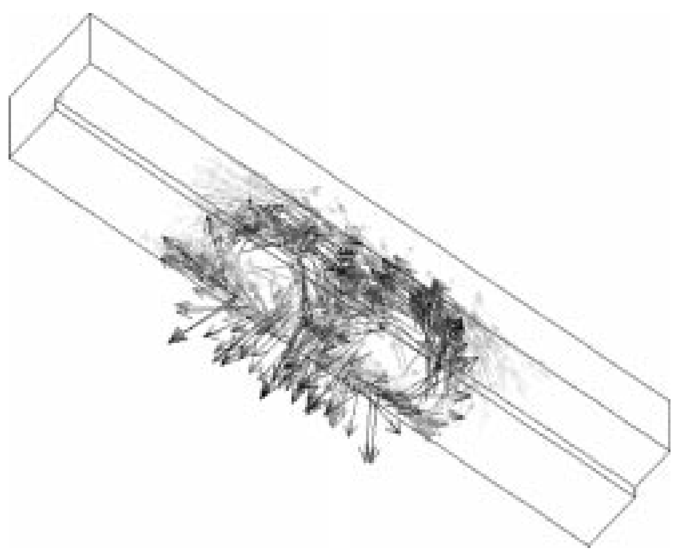

Fig. 7. Current density $\vec{J}$ in A/m^2 ($v = 1$ m/s).

V. Transient Eddy Current Problems

A. Formulation

The $\vec{A} - \vec{A}$, $\vec{T}$ formulation can be used for the solution of transient eddy current field problems as well. In contrast to the time-harmonic field problem (Fig. 2) any time progression of the fields can be taken into account by using the time stepping variant of the presented formulation displayed in Fig. 8. The nonlinearity of the iron parts can be considered by using the Newton–Raphson method. The transient equations to solve the

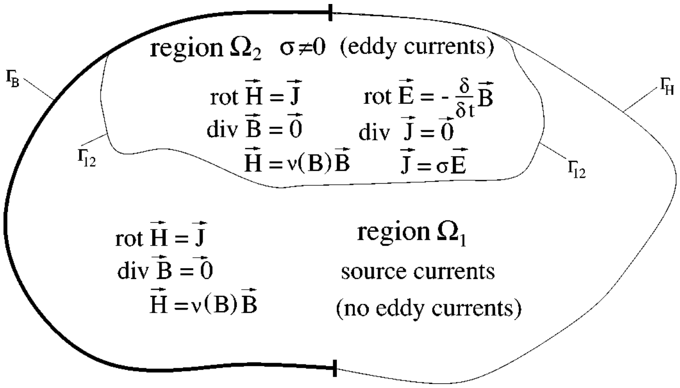

Fig. 8. Transient eddy current field problem.

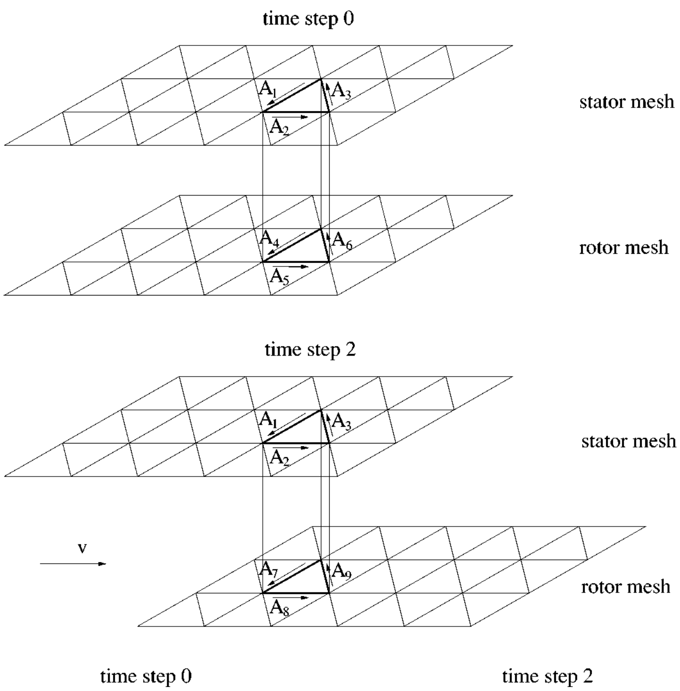

Fig. 9. Boundary constraints changing at each time step.

degrees of freedom for the magnetic vector potential $\vec{A}$ in the excitation region Ω_1 are given by:

$$\begin{aligned}&\int_\Omega \tau \text{curl}\,\vec{\alpha}_i\cdot\nu\text{curl}\,\vec{A}_{n+1}\,d\Omega\\&\quad=\int_\Omega((\tau-1)\text{curl}\,\vec{\alpha}_i\cdot\nu\text{curl}\,\vec{A}_n\\&\qquad+\tau\vec{\alpha}_i\cdot\vec{J}_{n+1}+(1-\tau)\vec{\alpha}_i\cdot\vec{J}_n+\text{curl}\,\vec{\alpha}_i\cdot\nu\vec{B}_r)\,d\Omega.\end{aligned}$$

In the eddy current region Ω_2 the time stepping variant of the $\vec{A},\vec{T}$ formulation has to be used:

$$\begin{aligned}&\int_\Omega(\tau\text{curl}\,\vec{\alpha}_i\cdot\nu\text{curl}\,\vec{A}_{n+1}-\tau\vec{\alpha}_i\cdot\text{curl}\,\vec{T}_{n+1})\,d\Omega\\&\quad=\int_\Omega((\tau-1)\text{curl}\,\vec{\alpha}_i\cdot\nu\text{curl}\,\vec{A}_n-(\tau-1)\vec{\alpha}_i\cdot\text{curl}\,\vec{T}_n)\,d\Omega\\&\int_\Omega\left(\tau\text{curl}\,\vec{\alpha}_i\cdot\frac{1}{\sigma}\text{curl}\,\vec{T}_{n+1}+\frac{1}{\Delta t}\text{curl}\,\vec{\alpha}_i\cdot\vec{A}_{n+1}\right)\,d\Omega\\&\quad=\int_\Omega\left((\tau-1)\text{curl}\,\vec{\alpha}_i\cdot\frac{1}{\sigma}\text{curl}\,\vec{T}_n+\frac{1}{\Delta t}\text{curl}\,\vec{\alpha}_i\cdot\vec{A}_n\right)\,d\Omega.\end{aligned}$$

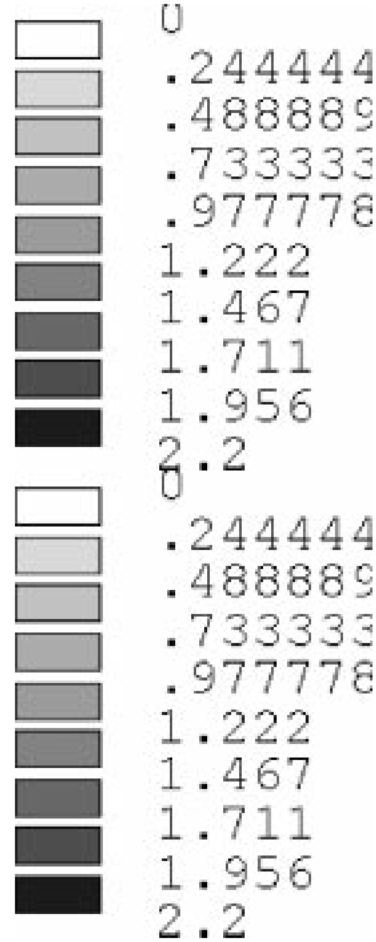

Fig. 10. Flux density distribution in T in the 81st time step.

Using the transient formulation the movement of conductors can be taken into account by moving the mesh of the conductor from time step to time step.

B. Application

With the presented 3D transient equations of the $\vec{A}-\vec{A},\vec{T}$-formulation it is possible to calculate the electromagnetic field distribution of an induction machine. The movement of the rotor is considered by turning the rotor at each time step. The disconnected stator and rotor meshes are joined by applying new boundary conditions on the contacting edges of these meshes after each time step, see Fig. 9.

Figs. 10 and 11 show the distribution of the magnetic flux density for two different time steps. Dynamic processes in the machine can be calculated by that method. The presented formulation considers as well the induced rotor slot currents as the nonlinearity of the iron and the real movement of the squirrel cage.

VI. Conclusion

In this paper the new edge based $\vec{A}-\vec{A},\vec{T}$ formulation is used for the calculation of different 3D eddy current field problems. Because of the advantages of this formulation to the well-known

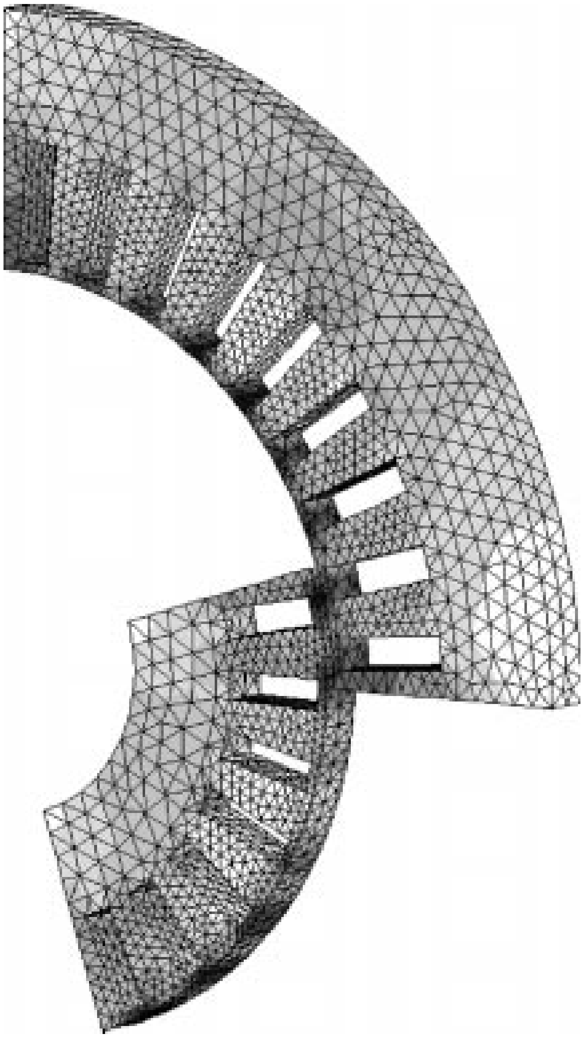

Fig. 11. Flux density distribution in T in the 89th time step.

$\vec{A}$, V formulation or $\vec{t}$, Φ formulation the $\vec{A} - \vec{A}$, $\vec{T}$ formulation is extended to eddy current field problems induced by motion and transient eddy current fields. Calculation examples and results for each of the presented Finite Element approaches are presented.

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