

# Three-dimensional Calculation and Optimization of the acoustic Field of an induction Furnace caused by electromagnetic Forces

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**Abstract**—This paper presents the three-dimensional electromagnetic, structural-dynamic and acoustic calculation of electric machines with respect to induction furnaces and gives hints for optimization of complex vibrating structures. The method chosen to compute the acoustic field is the Boundary Element Method, which solves the Helmholtz equation in the three dimensional case and considers scattering and symmetry of large scale structures [3]. The results presented in this paper show the advantage of BEM concerning calculation time and computation resources. The aim of the work is to optimize the acoustic behavior of the structure in such a way that noise emission of the machine will decrease. Several modifications are proposed and also calculated and show good results in comparison to the unmodified model.

**Index terms**—Induction heating, Acoustic noise, Boundary element methods, Electromagnetic forces, Finite element methods

## I. INTRODUCTION

The computation of complex structures such as electric machines or apparatus for induction heating requires the abstraction of the real existing structure. Recent developments in computer architecture made it possible to use also algorithms that generate full matrices during computation such as the Boundary Element Method (BEM). This approach is applied in an advantageous way if open boundary problems have to be solved, like the acoustic field of a radiating structure. In this paper the calculation of the coupling of electro-magnetically caused forces to the acoustic sound emission is presented considering an induction furnace in fully three-dimensional way [2]. Considering the forces, which cause the vibration of the structure, there are two basic components: Lorentz forces caused by the high coil currents in each eddy-current region of the structure and surface forces caused by the large differences of magnetic permeability at the cross section of the yokes as shown in Fig. 1. The approach used to calculate the magnetic field considers the time harmonic

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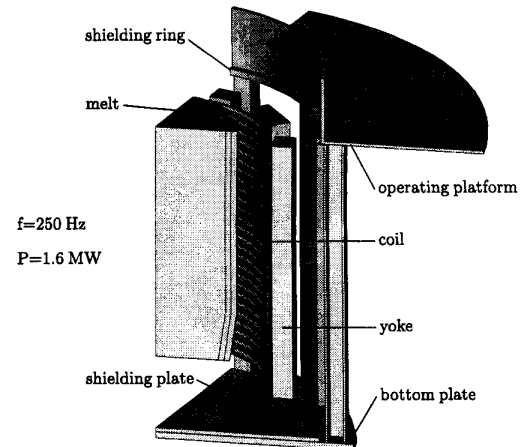


Fig. 1. The FE model for the 3D electromagnetic calculation

nature of the excitation and takes advantage of an edge-based vector potential approach [1].

## II. THE INDUCTION FURNACE

The principle of the melting apparatus is based on the induced eddy currents in the melting inset. The induced currents heat up the material due to the ohmic losses produced during the operation cycle. The magnetic field, which is necessary to the power transfer, is generated by a 26 turn coil. Due to the high currents in the coil of nearly 6.7 kA and the high induced eddy-currents, there are considerable losses and forces exist in these turns. In the following sections a coupled calculation of three different approaches is presented, which is based on Finite Element or Boundary Element mesh and local mesh refinement.

## III. THE FORCE CALCULATION

In this application there exists two different natures of forces caused by the electro-magnetic field. The largest amount of the resulting forces comes from the Lorentz-forces in regions of low electric resistance like the turns and the melt, as mentioned above. Regarding the yokes there exist magnetically caused surface forces due to the large change in magnetic permeability. The electro-magnetic field calculation is done by different approaches, which consider the dimensionality of the problem. For the

optimization of the Lorentz-forces in the furnace a 2D axisymmetric approach is chosen which uses the governing equation:

$$\text{curl} \nu \text{curl} \vec{A} + j\omega\sigma \vec{A} + \sigma \text{grad} V = 0, \quad (1)$$

where  $\vec{A}$  is the magnetic vector potential,  $V$  the electric scalar potential,  $\nu$  the reluctivity,  $\sigma$  the specific electric resistance and  $\omega$  the angular frequency of the current in the active coil. The Galerkin formulation results to [4]:

$$\int_{\Omega} \left( \nu \text{curl} \vec{N} \text{curl} \vec{A} + \vec{N} j\omega\sigma \vec{A} + \vec{N} \sigma \text{grad} V \right) = \vec{0}. \quad (2)$$

With respect to axisymmetric problems the equation can be written in axisymmetric coordinates, which have a singularity at the axis  $r = 0$ . This can be changed by applying the formula to a vector potential  $\vec{A}' = \vec{A} \cdot r$ .

For the calculation of the vibration a 3D time-harmonic  $A, T$ -approach using edge elements is applied, and uses the following vector potentials for the calculation [1]:

$$\vec{B} = \text{curl} \vec{A} \quad (3)$$

$$\vec{J} = \text{curl} \vec{T} \quad (4)$$

$$\text{curl} \nu \text{curl} \vec{A} = \text{curl} \vec{T} \quad (5)$$

$$\text{curl} \frac{1}{\sigma} \text{curl} \vec{T} = -j\omega \text{curl} \vec{A}. \quad (6)$$

In this case  $\vec{T}$  and  $\vec{A}$  are complex vector potentials which results in 6 variables per edge. The matrix is more sparse in comparison to node based solvers and this results in a smaller calculation time. Another advantage is the higher accuracy of edge elements.

#### A. Volume-forces in eddy-current regions

The Lorentz-forces can be calculated according to the following approach with regard to three dimensional problems:

$$\vec{F}(t) = \vec{J}(t) \times \vec{B}(t) \quad (7)$$

$$\begin{pmatrix} F_x(t) \\ F_y(t) \\ F_z(t) \end{pmatrix} = \begin{pmatrix} J_x(t) \\ J_y(t) \\ J_z(t) \end{pmatrix} \times \begin{pmatrix} B_x(t) \\ B_y(t) \\ B_z(t) \end{pmatrix}. \quad (8)$$

Due to the sinusoidal nature of the current and the magnetic induction respectively, one can write in complex notation regarding the oscillating part of the force

$$\vec{F} = \begin{pmatrix} J_y B_z - J_z B_y \\ J_z B_x - J_x B_z \\ J_x B_y - J_y B_x \end{pmatrix} \cdot e^{j2\omega t}. \quad (9)$$

The comparison of 2D and 3D calculation of the losses is shown in Fig. 2. The discretization is shown in Table I.

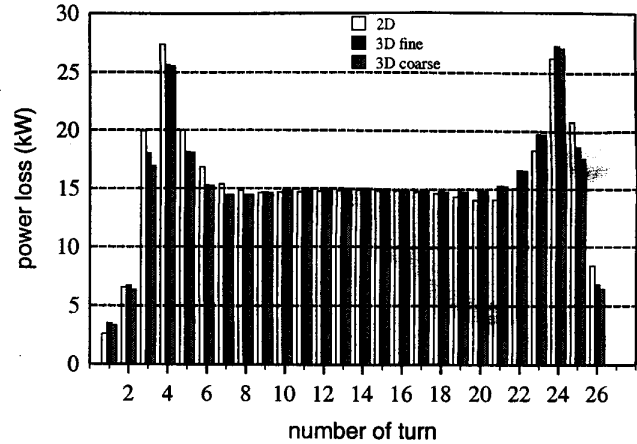


Fig. 2. Comparison of 2D and 3D loss calculation

TABLE I  
MESH DENSITY OF THE 2D AND 3D CALCULATION

	2D	3D coarse	3D fine
nodes	8188	22978	33271
elements	16282	117123	173556

The losses in the turns in the middle of the coil are nearly the same for all calculations, whereas at the end of the coil the losses in the 3D calculation are smaller than in 2D. The reason for this behavior can be the magnetic difference in circumferential direction and the discretization of the end turns.

#### IV. STRUCTURAL-DYNAMIC ANALYSIS

The structural analysis is done by applying Hamilton's principle to the equation of motion. The forces acting on a dynamic system are inertia forces, forces coming from the electromagnetic field, damping forces and stiffness forces. The theory of Hamilton postulates that the difference of

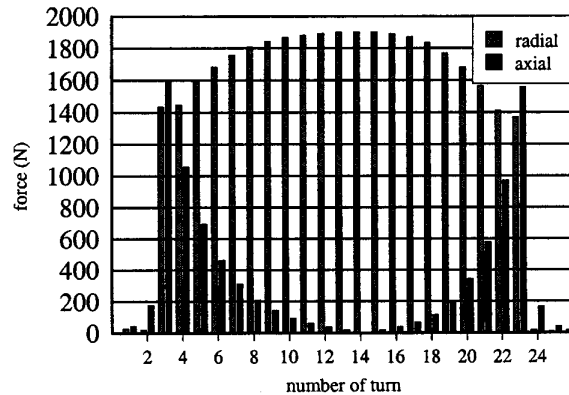


Fig. 3. Calculation of the 3D Lorentz force



Fig. 4. Real part of the deformation of the furnace (m)

kinetic and potential energy will result to a minimum in a dynamic mechanic system at equilibrium conditions. Considering a Finite Element approach one can write [6]

$$(\mathbf{K} - j\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{a} = \mathbf{F}. \quad (10)$$

The matrix  $\mathbf{N}$  corresponds with the interpolation matrix. The mass matrix contains the inertia forces of each element, which because of their efficacy act in the negative direction

$$\mathbf{M} = \sum_{i=1}^n \int_{V_i} \mathbf{N}^T \rho \mathbf{N} dV_i. \quad (11)$$

The potential energy is represented by the stiffness matrix, which contains the linear material property of the structure given by Hooke [5]:

$$\mathbf{K} = \sum_{i=1}^n \int_{V_i} \mathbf{B}^T \mathbf{H} \mathbf{B} dV_i. \quad (12)$$

The system of equation applied in this solver neglects the influence of material damping due to the small influence of the system and this leads to a uncoupled system for the real part and the imaginary part of the equation of motion. The result of the clauclation is shown in Fig. 4.

## V. ACOUSTIC FIELD CALCULATION

Acoustic problems normally have no closed region in which sound waves exist but a radiating structure emitting sound in the surrounding medium. The advantage of the boundary element method is shown in the discretization of problem. According to the following derivation only the vibrating structure has to be subdivided by boundary elements whereas the media is implemented by a special boundary condition, the Sommerfelds radiation

condition. The approach presented in this paper is the direct method, which means the use of *Greens theorem* for the solution of the three-dimensional Helmholtz function [3]. Fig. 5 shows the principal configuration of the acoustic problem, with  $\Gamma$  as a sufficient smooth surface of the object  $O$  and a surrounding region  $\Omega$  which is limited by a boundary  $\Gamma_\infty$  lying in infinity. The governing equation in the acoustic problem is the Helmholtz equation:

$$\Delta \underline{p} + k^2 \underline{p} = 0. \quad (13)$$

The method of the weighted residual with a weighting

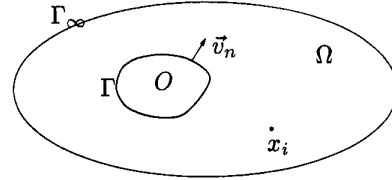


Fig. 5. Definition of the acoustic problem

function  $u^*$  is given by

$$\int_{\Omega} (\Delta \underline{p} + k^2 \underline{p}) u^* d\Omega = 0. \quad (14)$$

By applying Greens 2nd Theorem one gets a relation between the boundary  $(\Gamma + \Gamma_\infty)$  and the region  $\Omega$

$$\int_{\Omega} (\Delta \underline{p} + k^2 \underline{p}) u^* d\Omega = \int_{\Gamma + \Gamma_\infty} (u^* \frac{\partial \underline{p}}{\partial n} - \underline{p} \frac{\partial u^*}{\partial n}) d\Gamma + \int_{\Omega} (\Delta u^* + k^2 u^*) \underline{p} d\Omega. \quad (15)$$

The potential of sound  $p$  has to satisfy Sommerfelds radiation condition

$$\lim_{r \rightarrow \infty} \left( \frac{\partial \underline{p}}{\partial n} + ik \underline{p} \right) d\Gamma = 0, \quad (16)$$

$$\lim_{r \rightarrow \infty} \left( \frac{\partial u^*}{\partial n} + ik u^* \right) d\Gamma = 0. \quad (17)$$

Thus the boundary  $\Gamma_\infty$  gives no contribution to the problem. The fundamental solution in the three-dimensional case is

$$u^* = \frac{e^{-ikr}}{4\pi r}. \quad (18)$$

which is also conforming to the Sommerfeld condition. The discretization results to

$$c_i \underline{p}(i) + \sum_{j=1}^N \int_{\Gamma_j} \underline{p} \frac{\partial u^*}{\partial n} d\Gamma = -i\omega \rho \sum_{j=1}^N \int_{\Gamma_j} u^* \underline{v} d\Gamma. \quad (19)$$

The geometry factor is given by

$$c_i = \begin{cases} 0, & x_i \in O \\ \frac{\theta_i}{4\pi}, & x_i \in \Gamma \\ 1, & x_i \in \Omega. \end{cases} \quad (20)$$

The integration over each element is done by a quintic gaussian quadrature method with a transformation of singular integrals. Finally boundary conditions like symmetry and half-space are implemented.

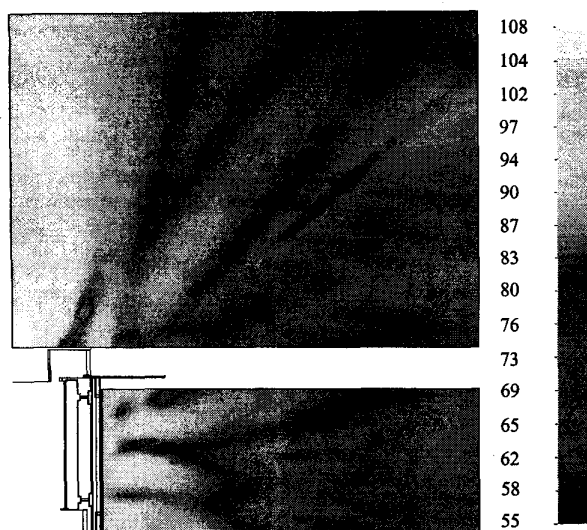


Fig. 6. Sound pressure of the actual model of the induction furnace (dB)

## VI. OPTIMIZATION

### A. Force optimization

In a next step a two-dimensional optimization is done using different shapes for the coil geometry, different air gaps and number of coils. An adaptive algorithm created a mesh up to 22.000 nodes in the final calculation was used to get a high accuracy for the Lorentz-force calculation. As shown in Fig. 7 the axial force has only 30 % of the total force. According to variant 5, which represents the initial shape, the radial and axial force component is reduced about 10 %, that is 1.2 kN in axial and 4.6 kN in radial direction.

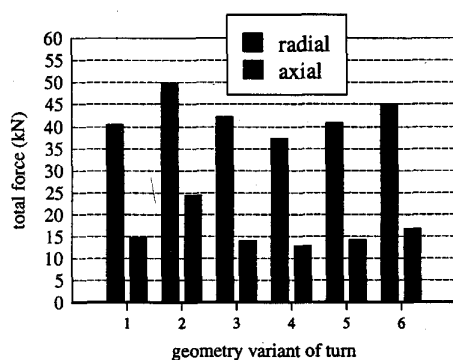


Fig. 7. Comparison of the total Lorentz-force of different turn shapes

### B. Structural optimization

The acoustic behavior of the induction furnace is similar to that of a piston membrane due to the large platform being excited in axial and radial direction. In order to

lower the amplitudes several modifications have been implemented in the structural-dynamic model. The result is shown in Fig. 8. Several variants are implemented and calculated, which are known in literature as the variation of mechanical impedance, the increase of stiffness or the additional mechanical coupling. Calculations show that increasing mechanical impedance has no great effect and tighter mechanical coupling of the vibrating parts to the outer construction increase the sound emission (variant 4 and 3). Some other mechanical variations achieve a reduction of sound pressure up to 10 dB (variant 2).

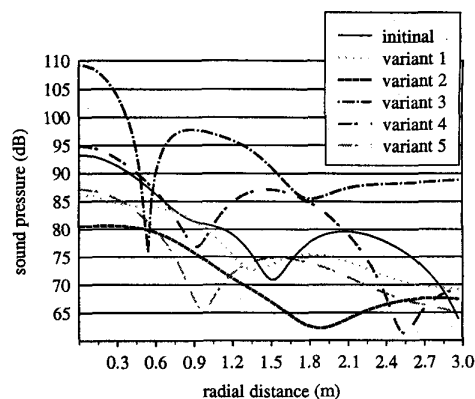


Fig. 8. Sound pressure on a radial contour over the platform using different geometry modifications

## VII. CONCLUSIONS

This paper presents the 3D calculation of an induction furnace, which is done by using FE and BE methods. The coupled calculation of forces, deflections and sound pressure is a suitable tool to reduce sound emission of rotating and static electric machines, like induction furnaces. The calculated reduction of sound pressure is 6 – 10 dB and the reduction of forces is 10 %.

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