# A Comparison of MRI Magnet Design using a Hopfield Network and the Optimized Material Distribution Method

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***Abstract*—Two approaches to device design, the Hopfield Network and Optimized Material Distribution (OMD), are compared. The comparison is based on a previously published MRI dipole magnet design problem. Since this problem is already formulated as a material distribution problem, the results focus on the optimization procedures used in the two methods, i.e. the learning algorithm of the Hopfield network versus the conjugate gradient algorithm used in OMD. Results of the two methods are presented, and are also compared to a previously published simulated annealing solution.**

***Index terms*—Optimization methods, Hopfield networks, Magnetic resonance imaging**

## I. Introduction

This paper compares the features of two methods of device synthesis. The first uses a Hopfield network, which has been applied to electromagnetic design previously [1], [2], although the approach used here is considerably different. The second is the Optimized Material Distribution (OMD) method, which uses the conjugate gradient method coupled with the Augmented Lagrange Multiplier method for handling constraints.

A simple design problem is used to compare the relative performance of each method, and in fact this test problem is set up such that both methods optimize the distribution of material (coil windings in this case).

This paper first introduces the test problem used to compare both methods. Then the Hopfield algorithm is outlined, followed by a brief description of the OMD method. Finally, results are presented, comparing both.

## II. Test problem

The test problem used to compare both methods is the optimization of the field homogeneity in a simple dipole magnet used for MRI. This problem was introduced in [3], where it was solved using simulated annealing techniques.

In this problem, coils are arranged around a square region. Each coil can be switched either *on* or *off*. The goal is to make the field as homogenous as possible at a set of sample points inside the square region.

This problem possesses several symmetries. Translational symmetry reduces the problem to 2-D. The left boundary is a plane of anti-symmetry (a homogeneous Dirichlet boundary condition), and the bottom boundary is a plane of symmetry (a homogenous Neuman boundary condition). Taking this symmetry into account, there are 180 independent coils, and 9 sample points.

In the final device, each coil must be either *on* or *off*, However, both methods described in this paper expand the search space to include the continuous states between these extrema. The requirement of a discrete final state is treated as a constraint.

## III. Hopfield neural networks

A Hopfield net is a fully connected neural network. Each neuron $Y_i$ has an internal activity $u_i$ and fires with $g(u_i)$ over a grid of weights $w_{i,j}$ to every neuron.

All information about the problem to be solved is given to the net in advance and expressed as an energy function, which the net tries to minimize [4]. The use of a predefined energy function implies that the problem must be superposable, so the effect of each neuron on the energy function can be calculated independently.

### A. The Hopfield energy function

The output signal of one neuron $v_i = g(u_i)$ corresponds to the current in the coil $i$. To obtain the desired output range $0 \leq v_i \leq 1$ the neurons have sigmoid monotonic input-output relations using $g(u_i) = \frac{1}{2}(1 + \tanh(u_i))$.

Two terms of the energy function are

$$E_x = \sum_j \left( \sum_i d_{x_{i,j}} v_i - H_{x_j} \right)^2 \frac{1}{H_d^2} \quad (1)$$

$$E_y = \sum_j \left( \sum_i d_{y_{i,j}} v_i - H_{y_j} \right)^2 \frac{1}{H_d^2}. \quad (2)$$

Where $d_{x_{i,j}}$ and $d_{y_{i,j}}$ are the effect of neuron $i$ on the $x$- and $y$-component of the magnetic field at test point $j$ derived from Maxwell's equations, and $H_{x_j}$ and $H_{y_j}$ are the desired values, e.g. the magnetic field. Therefore the minimization of $E_x$ and $E_y$ forces the components of the magnetic field towards the desired values.

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In the Hopfield net the energy function also includes the constraints. During the search the net is allowed to use invalid states, e.g. the continuous states between *on* and *off*. The inbetween states are actively discouraged through the use of a penalty function. The term

$$E_c = \sum_i v_i - \sum_i v_i v_i = \sum_i v_i(1 - v_i) \quad (3)$$

forces $v_i$ to either $v_i = 0$ or $v_i = 1$. After initializing the neurons, the Hopfield neural network tries to minimize the function $E = E_x + E_y + E_c$.

The Hopfield network tends to scatter the *on* conductors stochastically so they are not grouped together. To avoid this, an additional energy term is added which vanishes if the activity of one neuron $i$ equals the activity of its $n_i$ neighbors given as the list $N_i$:

$$E_f = \sum_i (\sum_{j \in N_i} v_j - n_i v_i)^2 \quad (4)$$

### B. Updating algorithm

In Hopfield neural networks the weights are not adjusted as the minimization proceeds but instead the neurons are changing their states, being updated asynchronously and randomly one at a time using the following updating formula for neuron $k$

$$u_k = u_k(old) + \Delta t(-u_k(old) - \Omega) \quad (5)$$

$$\Omega = \frac{\partial E_x}{\partial v_k} + \frac{\partial E_y}{\partial v_k} + \frac{\partial E_c}{\partial v_k} + \frac{\partial E_f}{\partial v_k} \quad (6)$$

$$\begin{aligned} &= 2\sum_j(\sum_i d_{x_{i,j}} v_i - H_{x_j}) d_{x_{k,j}} + (1 - 2v_k) \\ &\quad + 2\sum_j(\sum_i d_{y_{i,j}} v_i - H_{y_j}) d_{y_{k,j}} \quad (7) \\ &\quad - 2n_k(\sum_{j \in N_k} v_j - n_k v_k) + 2\sum_i(\sum_{j \in N_i} v_j - n_i v_i)\delta_i \end{aligned}$$

$$\delta_i = \begin{cases} 1 & \text{if } j = k \text{ for one } j \in N_b(i) \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

with $\Delta t$ as a factor controlling the learning rate in order to avoid oscillations of the process [6]. Through (7) the weights can be given to the net. The process terminates if the net is in a "freezing" state, i.e. the energy varies less than a threshold value. The investigations revealed that, indeed, the neurons converged very close to a state of either *on* or *off*.

## IV. Optimized Material Distribution

The Optimized Material Distribution (OMD) method is described in [7] and [8]. The essence of this approach is to represent devices as a distribution of material. This allows topologically different devices to be represented with the same set of parameters. Therefore an numerical optimization procedure can explore different device topologies. This is not possible when a device is represented by parameterizing its dimensions.

Note that the initial state can consist of free space (no material anywhere). The optimization will add material where it is required in order to achieve the design objectives. This eliminates the need for a template device on which to base the parameterization.

Because no template device is required, and because different device topologies can be explored within the same design space, OMD is actually doing design.

### A. Representation

The basis idea in OMD is to represent devices as a distribution of material. In practice, a *design region* is defined, which specifies the region which the device is allowed to occupy. This can be as simple as a rectangular region, although any information about areas required to be free of material can be excluded from the design region to simplify the objective function.

The design region is discretized into cells, and a set of parameters is assigned to each cell to control the amounts of various materials which will fill the cell. This representation can be compared to a graphical bitmap image of the device, where each "pixel" (cell) has a single "color" (material), but the net effect is to produce a more or less well-defined device topology. Note that during optimization, the material in a cell can vary continuously between empty and full (analogous to a grey-scale image).

The design region for the MRI test problem uses a relatively coarse discretization, since the distribution of material in this case consists of discrete coils.

### B. Objective function

The objective function is the same as the energy function for the Hopfield network, i.e. $E_x + E_y$. However, to minimize it the conjugate gradient algorithm is used. This requires the derivative of the objective function with respect to the material distribution. In this problem, this is easy to calculate since the magnetic field is a linear function of the current in each coil. However, note that the OMD method is not restricted to linear problems, since the derivative of general electromagnetic systems can be computed using the adjoint variable method of sensitivity analysis.

### C. Solidification Constraint

The constraint term forces each cell to contain only one material, in other words to be *solid*. In this context, free space is also considered to be a solid. For the MRI test problem, this is equivalent to forcing each cell to be either on (100% current) or off (zero current).

The constraint term is enforced gradually, by using the Augmented Lagrange Multiplier (ALM) method. This method performs a series of un-constrained minimizations of the original objective function augmented with penalty terms. Each unconstrained minimization is performed using the conjugate gradient method. In this case the penalty terms are functions of $E_c$ defined above:

$$F_ALM = E_x + E_y + \lambda_i E_c + w_i E_c^2 \quad (9)$$

where $w_i$ is the weight and $\lambda_i$ the Lagrange mupltiplier at the i-th ALM iteration. At the end of each ALM iteration, the Lagrange multiplier is updated to $\lambda_{i+1} = \lambda_i + 2w_iE_c$.

The algorithm is started with the Lagrange multiplier set to zero and the weight chosen to make the cost and the penalty terms approximately equal. For the MRI test problem the initial penalty weight is set to 0.1.

## V. Results

Both algorithms are applied to the MRI test device, with the following results. Fig. 1 shows the final state found by the Hopfield net after 2000 updates of each neuron. The solid and blank squares represent the *on* and *off* state respectively, with the current in each coil set to $I = 1$ A. The desired magnitude of the magnetic field at the 9 testpoints is $H_x = 0$ and $H_y = 36.25811$ A/m, which are the values calculated from the coil distribution in [3].

Fig. 2 shows the behavior of the Hopfield network during the minimization of the energy function $E$ and its components. Note that the net reduces the values of $E_x$ and $E_y$ first and then minimizes $E_C$ to force the coils to be either *on* or *off*. Table I summarizes the energy values after 200 and 2000 epochs in the first three columns. The ($D$) columns inform about the final state with all neurons set to a discrete value. A snapshot after 200 epochs is shown in Fig. 3 where the area of the squares is proportional to the current. Some units are fully developed while other are in an intermediate state.

Adding the $E_f$ term to avoid stochastic output results in the current distribution displayed in Fig. 4. The energy values are shown in the column 5 and 6 in Table I. The final total energy surprisingly converged to a lower value.

Fig. 6 shows the final state found by OMD after 10 ALM iterations of 10 conjugate gradient iterations each.

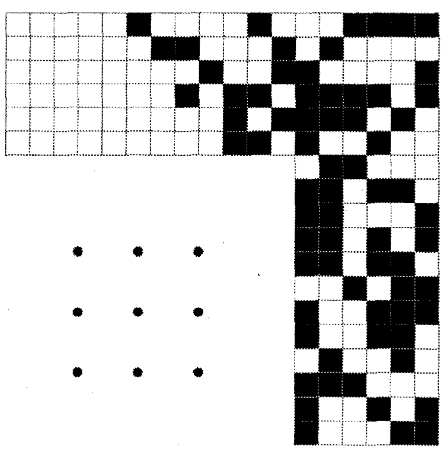

Fig. 1. The final state of conductors produced by the Hopfield net

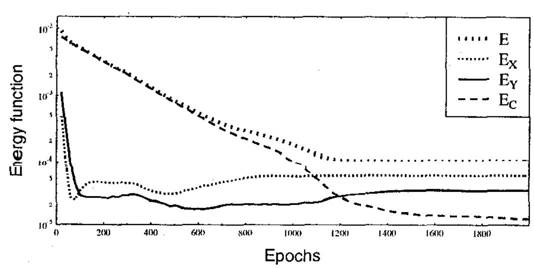


Fig. 2. The energy function during the minimization process

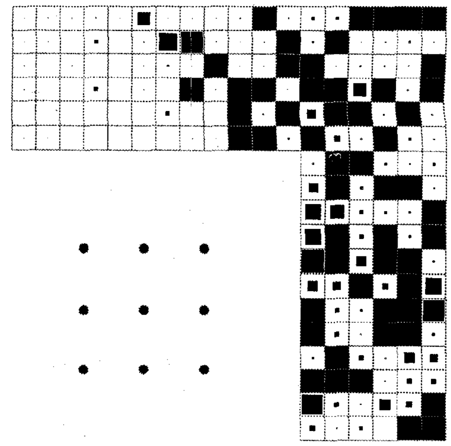

Fig. 3. A snapshot of the currents in the conductors (Hopfield net)

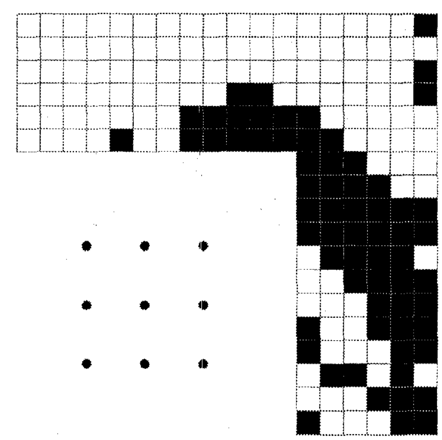

Fig. 4. Alternative final state produced by the Hopfield net using the neighbor penalty term

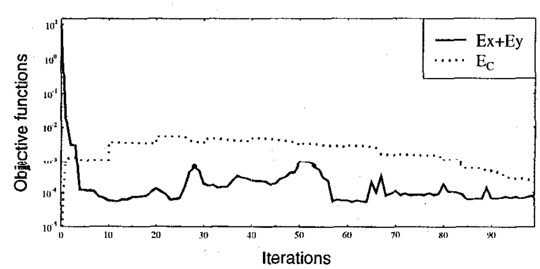


Fig. 5. The energy function during the minimization process (OMD)

TABLE I Comparison of the different methods with energy functions

| Method | Hopfield neural network | | | Hopfield using (4) | | OMD method | | S&T[3] |
|---|---|---|---|---|---|---|---|---|
| Iterations | 200 | 2000 | 2000 | 1100 | 1100 | 100 | 100 | |
| Continuous/Descrete | C | C | D | C | D | C | D | D |
| $E_x + E_y$ | 6.7590-5 | 8.9106e-5 | 1.0573e-4 | 6.9111e-5 | 9.606e-5 | 0.858e-4 | 1.501e-4 | 0.867e-4 |
| $E_c$ | 7.3901e-3 | 2.8398e-5 | 0 | 5.3033e-3 | 0 | 2.407e-3 | 0 | 0 |
| $E_f$ | - | - | - | 0.29247 | 0.5598 | - | - | - |

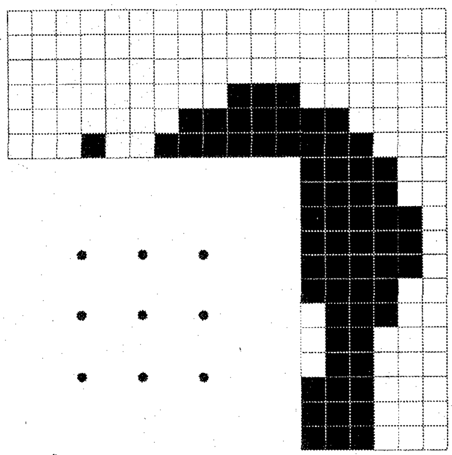

Fig. 6. The final state of conductors produced by OMD

The penalty weight was equal to 0.1 and was constant throughout, and the final value of the Lagrange multiplier was 0.0905. Fig. 5 shows the field homogeneity and solidification constraint value throughout the optimization.

## VI. Discussion

Several additional experiments were performed in an effort to understand these results.

Sampling the field at more points resulted in essentially the same homegeneity value, indicating that 9 sample points is sufficient.

More insight is gained by trying to determine the nature of the continuous search space between different coil configurations. One way of doing this is to take a point in the search space midway between the Hopfield and OMD solutions (i.e. by averaging the currents in each coil). It turns out that, while this point does not satisfy the discreteness constraint, the homogeneity of this point is better than either the Hopfield or the OMD solution. In fact, this is expected since the field is a linear superposition of the fields produced by each coil, so any linear combination of highly homogeneous solutions must also be highly homogeneous, and in fact the errors will tend to cancel out, so the homogeneity will be better (i.e. all configurations are in the same basin of attraction in the continuous search space). Of course, this doesn't help to find the optimal solution, since these intermediate configurations do not have discrete coil states, but it does give an idea of the nature of the continuous search space.

## VII. Conclusion

The paper has provided a comparison of two optimization approaches for a "standard" benchmark.

The basis of comparison between the two methods lies in how well they generate configurations in which the *on* coils are grouped together. In this respect, both the original simulated annealing solution and the Hopfield network without special constraints performed poorly. However, with the neighbor penalty term the Hopfield network achieved some coil grouping with a homogeneity superior to that achieved with OMD. OMD did have a slightly better coil grouping, which may have more to do with the way the solidification constraints (to force a discrete final state) were enforced than with the stochastic nature of the Hopfield optimization.

The results show that both achieve similar results but the cost of the Hopfield is much less than the cost of OMD. However OMD is more general since no added terms were required to produce a grouped result.

## VIII. Acknowledgements

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