A Sensitivity approach for the Optimization of Loss Efficiencies

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***Abstract*— This paper presents the optimization of the loss efficiency of inductive heating devices by the application of a sensitivity approach. Based on a mixed potential FE skin effect problem a new sensitivity formulation is derived and extended to the adjoint variable method. Through this the amount for the calculation of the cost function and its gradient is minimized. The algorithm is applied to the positioning of the field turns of a simple inductive heating device. Finally the optimization of a crucible induction furnace is shown.**

I. Introduction

The optimization of the efficiency of eddy current devices in the field of induction heating is one of the basic demands to the designing process. Since numerical field calculations are employed more frequently, the combination with optimization techniques has become very important.

Very popular are stochastic methods (simulated annealing, evolutionary strategy and genetic algorithms) where standard field calculation packages can be employed. The implementation of boundary conditions is easy and the global optimum will reliably be found. The major drawback is the quite extense number of field calculations needed to find the optimum. Here the use of gradient methods seems very promissing to reduce the computational amount [1]. Though these methods can be trapped in local optima, their application needs some special care.

The gradient information for these methods is usually gained by the sensitivity analysis. The sensitivity analysis with respect to eddy current problems is a quite new research subject. [2] shows a minimum volume analysis of a magnetic shield. In [3] a shape optimization is done to achieve a defined flux density distribution. [4] gives a formulation for the so called continuum approach using the adjoint variable method.

In this paper an efficient FE skin effect sensitivity formulation for the loss efficiency is given. It is extended to the adjoint variable method. This allows the computation of the cost and its complete gradient with just two solutions of a linear set of equations.

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II. Getting the gradient

In this section the sensitivity formula for a linear skin effect problem will be deduced.

A. Sensitivity analysis

ψ is a cost function with the $(m \times 1)$-vector p of m design parameters

$$p = \{p_1, p_2, \ldots, p_m\}^T \tag{1}$$

and the $(n \times 1)$-vector X the solution for the skin effect field problem with n unknowns

$$X = \{X_1, X_2, \ldots, X_n\}^T \tag{2}$$

$$\psi = \psi(p, X) \quad \text{with} \quad X = X(p) \tag{3}$$

The l-th component of the total differential of ψ according to the design parameter p_l is

$$\frac{\mathrm{d}\psi}{\mathrm{d}p_l} = \frac{\partial \psi}{\partial p_l} + \frac{\partial \psi}{\partial X}^T \cdot \frac{\partial X}{\partial p_l} \tag{4}$$

The linear matrix equation for the field problem is

$$K X = b \tag{5}$$

From this the $(n \times 1)$-vector $\partial X / \partial p_l$ can be derived by

$$K \frac{\partial X}{\partial p_l} = -Y \tag{6}$$

with the $(n \times 1)$-vector Y

$$Y = \frac{\partial}{\partial p_l}(K X) - \frac{\partial b}{\partial p_l} = \frac{\partial}{\partial p_l}(K X - b) \tag{7}$$

The right side Y is for each variable a known vector, that can be computed from the knowledge of the FE-model and the solution X. The problem of finding $\frac{\partial X}{\partial p}$ is then simply reduced to another solution of a linear set of equations with the same matrix K that has been used to compute X. From this the calculation of $\frac{\mathrm{d}\psi}{\mathrm{d}p}$ is straight forward and simple in many cases. To get the cost and its gradient,

$m+1$ matrix solutions are necessary. This can be too expensive in most application cases.

The adjoint variable method is a tool to compute the same results by just two matrix solutions. We choose the $(n \times 1)$-adjoint variable λ for the solution of the adjoint equation

$$K\,\lambda = \frac{\partial \psi}{\partial X} \tag{8}$$

With the symmetry of K we get

$$\frac{\partial \psi}{\partial X}^T \frac{\partial X}{\partial p_l} = (K\lambda)^T \frac{\partial X}{\partial p_l} = \lambda^T K \frac{\partial X}{\partial p_l} = -\lambda^T Y \tag{9}$$

Then the sensitivity of the l-th component of ψ simply is

$$\frac{\mathrm{d}\psi}{\mathrm{d}p_l} = \frac{\partial \psi}{\partial p_l} - \lambda^T\,Y \tag{10}$$

B. Cost function

To gain a design of maximum loss efficiency, losses in the field coil P_{coil} have to be reduced by increasing the joule power in the workpiece P_w i.e. the objective function ψ is the ratio

$$\psi = \frac{P_{coil}}{P_w} \to \min \tag{11}$$

Its sensitivity with respect to the solution X can be written as

$$\frac{\mathrm{d}\psi}{\mathrm{d}X} = \frac{1}{P_w}\frac{\mathrm{d}P_{coil}}{\mathrm{d}X} - \frac{P_{coil}}{P_w^2}\frac{\mathrm{d}P_w}{\mathrm{d}X} \tag{12}$$

In this paper the minimization will be achieved by a change of the spatial position of the exciting coils and not by a shape variation although the presented sensitivity approach is not restricted to this.

C. The skin effect field problem

As the computation of the field (5) and the sensitivity (8) works on the same matrix K, an efficient formulation for the 2D skin effect problem has to be chosen. The incorporation of the total current according to the integrodifferential approach [2] causes slow and costly computations because the matrix is relatively dense. In this paper a mixed potential formulation was chosen preserving the sparsity and the symmetry of the matrix [5]. (5) becomes

$$\begin{pmatrix} \nu\,S + \mathrm{j}\omega\sigma\,T & -\sigma\,C \\ -\sigma\,C & \dfrac{\sigma\,F}{\mathrm{j}\omega} \end{pmatrix} \cdot \begin{pmatrix} A \\ U \end{pmatrix} = \begin{pmatrix} 0 \\ \dfrac{I}{\mathrm{j}\omega} \end{pmatrix} \tag{13}$$

with the angular frequency ω, I its driving current, σ the conductivity, ν the reluctivity and for the field solution the vector of the magnetic vector potential A and the voltage vector U. F is the cross-sectional area of the device. The submatrices S, T and C can be calculated from

$$S_{ij} = \int_\Delta \nabla N_i\,\nabla N_j\,|G|\,\mathrm{d}u\mathrm{d}v \tag{14}$$

$$T_{ij} = \int_\Delta N_i\,N_j\,|G|\,\mathrm{d}u\mathrm{d}v \tag{15}$$

$$C_i = \int_\Delta N_i\,|G|\,\mathrm{d}u\mathrm{d}v \tag{16}$$

and the Jacobian matrix G. Using the current density

$$J = \sigma\,U - \mathrm{j}\omega\sigma A \tag{17}$$

and its sensitivity

$$\frac{\partial J}{\partial p_l} = \sigma\frac{\partial U}{\partial p_l} - \mathrm{j}\omega\sigma\frac{\partial A}{\partial p_l} \tag{18}$$

the loss can be calculated by

$$P = \frac{1}{\sigma}\sum_i\sum_j T_{ij}\,J_i\,J_j^* \tag{19}$$

D. Sensitivity for the loss efficiency

The loss sensitivity is

$$\frac{\partial P}{\partial p_l} = \frac{1}{\sigma}\sum_i\sum_j \left(\frac{\partial T_{ij}}{\partial p_l} J_i\,J_j^* + T_{ij}\frac{\partial(J_i\,J_j^*)}{\partial p_l}\right) \tag{20}$$

Though no shape distortion is considered the first part of the sum disappears.

$$\frac{\partial P}{\partial p_l} = \frac{1}{\sigma}\sum_i\sum_j T_{ij}\left(J_i\frac{\partial J_j^*}{\partial p_l} + J_j^*\frac{\partial J_i}{\partial p_l}\right) \tag{21}$$

$$= I_1 + I_2 \tag{22}$$

Expressing this sum in terms of the solution vector gives

$$I_1 = \sum_i\sum_j T_{ij}\,J_j\,(\frac{\partial U_i^*}{\partial p_l} + \mathrm{j}\omega\frac{\partial A_i^*}{\partial p_l}) = \frac{\mathrm{d}P}{\mathrm{d}X^*}^T\frac{\partial X^*}{\partial p_l} \tag{23}$$

$$I_2 = \sum_i\sum_j T_{ij}\,J_j^*\,(\frac{\partial U_i}{\partial p_l} - \mathrm{j}\omega\frac{\partial A_i}{\partial p_l}) = \frac{\partial P}{\partial X}^T\frac{\partial X}{\partial p_l} \tag{24}$$

The problem is now to operate not only with the sensitivity of the solution vector but also with its conjugate complex value which presently is not available. This could be overcome by solving the field problem together with its conjugate complex problem.

$$\begin{pmatrix} K & 0 \\ 0 & K^* \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \dfrac{\partial\psi}{\partial X} \\ \dfrac{\partial\psi}{\partial X^*} \end{pmatrix} \tag{25}$$

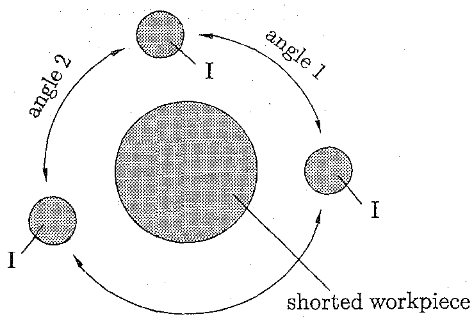

Fig. 1. Geometry of the optimization example

But it can easily be shown, that (23) is conjugate complex to (24) resulting in $\lambda_2 = \lambda_1^*$.

The first term in (10) is zero, because the cost function is not a geometrical quantity. So the sensitivity is

$$\frac{\mathrm{d}\psi}{\mathrm{d}p_l} = -(\lambda_1\,\mathrm{Y} + \lambda_1^*\,\mathrm{Y}^*) = -2\,\mathrm{Re}\,\{\lambda_1\,\mathrm{Y}\} \qquad (26)$$

As the submatrices in (25) are not coupled, λ_1 can be computed from (5), (12) combined with (24) and

$$K\,\lambda_1 = \frac{\partial\psi}{\partial X} \qquad (27)$$

Still unknown for the moment is Y. It can be taken from [3].

E. Optimization procedure

(26) gives the gradient information necessary for the optimization procedure. Here the conjugate gradient method with gradient projection technique was applied [6]. Cubic polynomial approximation and a bracketing technique with sequential quadratic interpolation was used for the one-dimensional search [7][8].

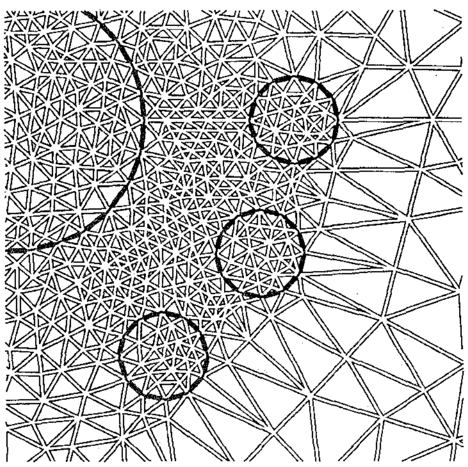

Fig. 2. Mesh for optimum geometry

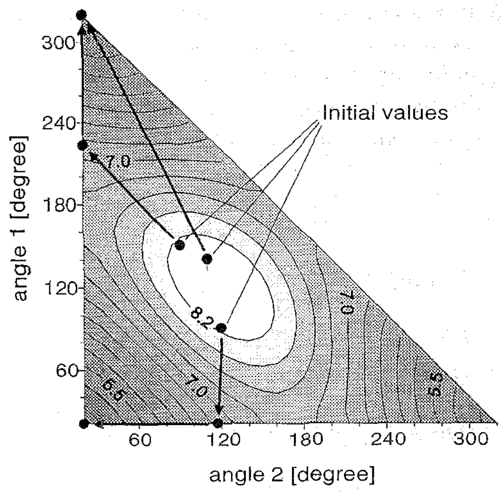

Fig. 3. Design space and optimization progress

III. Application

The decribed method is applied to the simple skin effect problem of Fig. 1 with two design parameters. This makes it possible to display the design space and to evaluate the performance of the optimization procedure. In a second application example results are shown for a coil distance optimization of an induction furnace.

A. Simple inductive heating device

Three cylindrical coils, each carrying the current I, are placed around a shorted conductor. The shape of all conductors is kept constant thoughout the optimization process. Only the position of the outer coils can be changed. For symmetry reason the right coil is fixed. The other two are free to move in the circumferential direction around the inner cylinder and to arrange in their optimal distances. The two distance angles are the design parameters. Their sensitivity is calculated from the three sensitivities of the coils. The modeling parameters

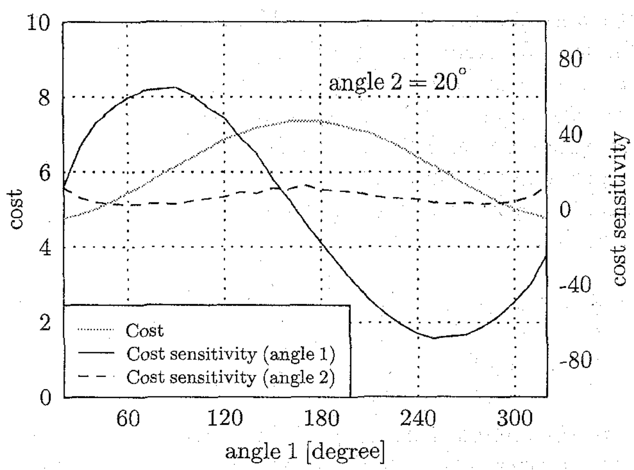

Fig. 4. Cost function and sensitivity

are $r_i = 15$ mm, $r_a = 31.83$ mm, $r = 5$ mm, $f = 100$ Hz and $I = 500$ A.

The minimum cost and therefore maximum efficiency is reached for the minimum coil distance which is fixed to 20 degrees. One optimal design is displayed in Fig. 2. Its mesh contains 2331 first order elements with 1186 nodes. The optimum is reached in the three corners of the design space (Fig. 3). The optimization process is given by pathes through the design space for different initial designs. Equidistant arranged coils (120 degrees for both) give minimum efficiency (but also minimum coil loss). The design space is bounded by the following linear constraints

$$\text{angle}\,1 \geq 20^\circ, \quad \text{angle}\,2 \geq 20^\circ, \quad \text{angle}\,1 + \text{angle}\,2 \leq 340^\circ$$

Fig. 4 shows the cost and the sensitivity on a slice through the design space. The sensitivity curve is smooth enough for gradient based optimization. Each optimum is reached after 8 FE function evaluations and 2 gradient computations i.e. 10 solutions of a linear set of equations.

B. Induction furnace

As a second application the 18 gaps between the 19 copper turns of a $4\,t$ steel crucible induction furnace are optimized. As boundary conditions the minimum gap and the maximum coil length are given. The turns move in the vertical direction to positions, where a maximum efficiency is reached. Fig. 5 shows the upper part of the model. The efficiency has been improved from 82.4 % for an equidistant distributed coil to 82.8 %. Especially the gap at the upper end of the coil has been enlarged due to the melt that protudes the coil (Fig.6).

The optimization of this model with 18 design parameters took 44 FE function evaluations and 6 gradient computations i.e. 50 solutions of a linear set of equations.

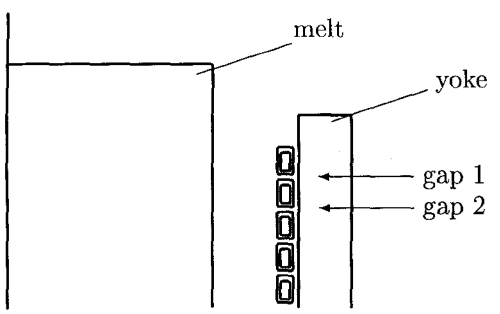

Fig. 5. Upper part of the model of the induction furnace

IV. Conclusion

The new formulation for the sensitivity approach for loss efficiencies combined with the adjoint variable method gives a powerful and fast tool for the design optimization process. The amount for the computation of gradients is minimized especially for a high number of design parameters. Therefore the application of efficient gradient based optimization techniques is possible.

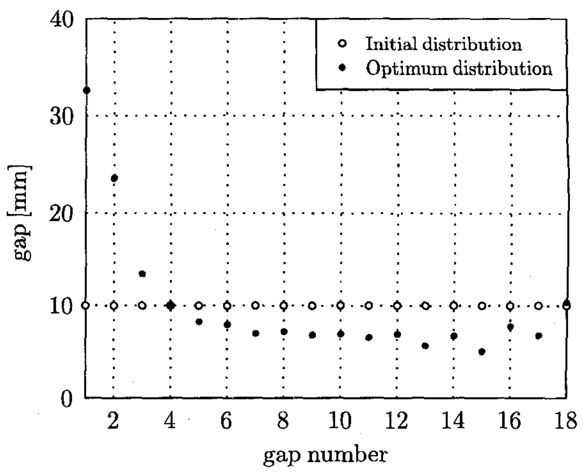

Fig. 6. Gaps for the initial model and for the optimum

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