F. Van De Meulebroeke

Senior Member, IEEE

Electrabel

Belgium

B-1000 Brussels

Direct Calculation of the Stability Domains of Three-Phase Ferroresonance in Isolated Neutral Networks with Grounded-Neutral Voltage Transformers

N. Janssens
Member, IEEETh. Van Craenenbroeck
Non MemberD. Van Dommelen
Senior Member, IEEELaborelecElectrical Engineering DepartmentB-1630 LinkebeekKatholieke Universiteit LeuvenBelgiumB-3001 LeuvenBelgiumBelgium

Abstract - Stability domains of ferroresonant oscillations must be computed and, if needed, countermeasures must be taken in order to avoid serious damage to the power networks' equipment. In this paper, Clarke components are used to write the equations of a representative three-phase circuit under a canonical form and give a simple physical interpretation of the three-phase ferroresonance phenomenon. A method is proposed to approximate the quasi-periodic oscillations by periodic ones. The solutions and their domains of stability are computed by the harmonic balance method. An application to a field case is presented.

1. Introduction

The ferroresonance phenomenon in power networks can cause dangerous overvoltages and overcurrents, which can lead to serious damage to the equipment. Therefore, it has to be avoided by all means. The cause of the phenomenon is an oscillation between a capacitance and a nonlinear inductance. Different types of ferroresonant oscillations occur, depending on the configuration of the network. An overview of possibly dangerous network configurations is given in [1]. All of them feature the existence of different stable solutions, for the same network parameters. The initial conditions, and events such as short circuits, determine which of these solutions will be attained.

95 SM 420-0 PWRD A paper recommended and approved by the IEEE Transmission and Distribution Committee of the IEEE Power Engineering Society for presentation at the 1995 IEEE/PES Summer Meeting, July 23-27, 1995, Portland, OR. Manuscript submitted December 21, 1994; made available for printing June 22, 1995. The ferroresonant oscillation is said to be single-phase, if the three-phase network can be reduced to a one-phase representation. I.A.Wright [2] has published experimental results and physical explanations of the influence of the grounding on this representation. A comprehensive study of single-phase ferroresonance is found in [3], where a method has been developed to compute directly the domain in some parameter space where ferroresonance can occur. The introduction of a detailed core model has given computation results in good accordance with full scale tests [4].

In electrical networks with isolated neutral, it is not possible to make a reduction to a single phase network. Typical configurations of isolated networks where threephase ferroresonance has been observed are power plants auxiliaries, distribution networks in factories, public distribution networks temporarily isolated.

Although the methods described in this paper could be applied to systems with three-phase transformers (magnetic coupling between the phases), we will only consider the simple configuration shown on fig.1. The three-phase voltage supply $\{U_1, U_2, U_3\}$ is considered to be balanced, C_0 is the zero sequence capacitance of a feeding cable and T is an inductive voltage transformer (V.T.).

A commonly used means to avoid three-phase ferroresonance consists in forming a delta connection with the tertiary windings of the V.T. closed on a damping resistance [5]. The aim of this paper is to describe an efficient method to determine if there is a risk of ferroresonance and to compute the damping circuit in order to avoid the phenomenon.



Fig.1. Network with isolated neutral

0885-8977/96/\$05.00 © 1995 IEEE

2. Models and Equations

In the study presented here, the model of the voltage transformer is chosen very simple and consists of a series resistance R_s , a shunt resistance R_p to represent the iron losses, the nonlinear magnetization characteristic $i(\phi)$ and an ideal transformer which represents the magnetic coupling with a tertiary winding. The modelling of the nonlinear inductance is essential to study ferrorescnance. The examples described below relate to a 6.6 kV V.T. The least-squares identification of the saturation characteristic from the available data led to a fifth-order polynomial:

$$\mathbf{i}(\phi) = \mathbf{k}_1 * \phi + \mathbf{k}_5 * \phi^5 \tag{1}$$

The three-phase network with tertiary winding and damping resistance R_d is shown on fig.2. The three capacitances C_0 have been replaced by one capacitance 3 C_0 placed between both neutrals. It can easily be proven that this does not alter the waveforms at the terminals of the nonlinear elements.

This network can be described by a system of four simultaneous differential equations (2-3) in the state variables { ϕ_1 , ϕ_2 , ϕ_3 , u_n }:

$$\begin{bmatrix} 1 + \frac{R_{s}}{R_{p}} + \frac{R_{s}}{R_{d}} & \frac{R_{s}}{R_{d}} & \frac{R_{s}}{R_{d}} \\ \frac{R_{s}}{R_{d}} & 1 + \frac{R_{s}}{R_{p}} + \frac{R_{s}}{R_{d}} & \frac{R_{s}}{R_{d}} \\ \frac{R_{s}}{R_{d}} & \frac{R_{s}}{R_{d}} & 1 + \frac{R_{s}}{R_{p}} + \frac{R_{s}}{R_{d}} \end{bmatrix} * \begin{bmatrix} \frac{d\phi_{1}}{dt} \\ \frac{d\phi_{2}}{dt} \\ \frac{d\phi_{3}}{dt} \end{bmatrix} \\ = \begin{bmatrix} u_{1}(t) - R_{s} \cdot i_{1}(\phi_{1}) - u_{n} \\ u_{2}(t) - R_{s} \cdot i_{2}(\phi_{2}) - u_{n} \\ u_{3}(t) - R_{s} \cdot i_{3}(\phi_{3}) - u_{n} \end{bmatrix}$$
(2)

$$(\frac{1}{R_{p}} + \frac{3}{R_{d}})(\frac{d\phi_{1}}{dt} + \frac{d\phi_{2}}{dt} + \frac{d\phi_{3}}{dt}) - 3C_{0}\frac{du_{n}}{dt} = 0$$
(3)

In order to write this system of equations under canonical form :

$$\frac{d\bar{x}}{dt} = \bar{F}(\bar{x},t) , \qquad (4)$$

we use the normalized Clarke transformation :



Fig.2 Full network representation

$$\begin{bmatrix} \mathbf{v}_{1} \\ \mathbf{v}_{2} \\ \mathbf{v}_{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & \mathbf{0} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} * \begin{bmatrix} \mathbf{v}_{0} \\ \mathbf{v}_{\alpha} \\ \mathbf{v}_{\beta} \end{bmatrix}$$
(5)

where v = u, i or ϕ . The system (2-3) is then rewritten in the variables { ϕ_0 , ϕ_{α} , ϕ_{β} , u_n }, each of the currents i_0 , i_{α} , i_{β} being a function of the three flux variables :

$$\begin{bmatrix} 1 + \frac{R_s}{R_p} + \frac{3R_s}{R_d} & 0 & 0\\ 0 & 1 + \frac{R_s}{R_p} & 0\\ 0 & 0 & 1 + \frac{R_s}{R_p} \end{bmatrix} * \begin{bmatrix} \frac{d\phi_0}{dt}\\ \frac{d\phi_\alpha}{dt}\\ \frac{d\phi_\beta}{dt} \end{bmatrix}$$
$$\begin{bmatrix} u_0 - R_s \cdot i_0 - \sqrt{3} u_n \\ 0 & 0 \end{bmatrix}$$
(6.a)

$$\begin{bmatrix} u_{\alpha} - R_{s} \cdot i_{\alpha} \\ u_{\beta} - R_{s} \cdot i_{\beta} \end{bmatrix}$$
 (6.c)

$$i_{o} + (\frac{1}{R_{p}} + \frac{3}{R_{d}}) \frac{d\phi_{o}}{dt} - \sqrt{3} C_{o} \frac{du_{n}}{dt} = 0$$
 (7)

with the Clarke components of the voltage supply given by

$$u_0 = 0$$
 $u_\alpha = U \sin (2 \pi f_0 t)$ $u_\beta = -U \cos (2 \pi f_0 t)$ (8)

where U is the rms line voltage of the supply, and taking as phase reference the positive passage through zero of phase 1. From (6) and (7), the canonical form (4) of the system of equations is easily obtained.

Let us remark that, in the case of different parameters in the three phases, a transformation dependent on these parameters ought to be used to reduce the system of equations into canonical form ; the Clarke transformation could not be used to this purpose in that case.

3. Physical interpretation

The expression of the circuit equations using the Clarke components allows to give a simple physical interpretation of the three-phase ferroresonance phenomenon : equations (6.a) and (7) describe the behaviour of the o component circuit shown on fig. 3.a (identical to the zero sequence circuit) ; equation (6.b) describes the behaviour of the α component circuit shown on fig. 3.b ; in the same way, equation (6.c) corresponds to the β component circuit shown on fig. 3.c.

If the inductances are linear, these three circuits are independent. Any transient will damp in such a way that the oscillation in the o component circuit will vanish and that the permanent evolution in the α and β circuits will be purely sinusoidal at the frequency of the voltage source.

If the inductances are nonlinear, these elements introduce a coupling between the three circuits. So, under certain circumstances, energy may be transfered from circuits α and β , where there is a voltage source, to the o component circuit and sustain a permanent oscillation. The frequency of this oscillation will be equal to or near a multiple or submultiple of the frequency of the voltage source. The amplitude of this o component oscillation will adjust itself in such a way that the resonance frequency of the circuit is one of the frequencies mentioned above. Which oscillation mode is reached depends upon the initial conditions.

The damping resistance R_d only appears in the o component circuit. Such a resistance, if carefully designed, may damp the o component oscillations and avoid any sustained ferroresonance.



4. Oscillation modes

The various oscillation modes that can appear in the circuits considered here were already described in the literature [5,6] and are listed in the table hereafter.

oscillation mode	symbol	main component
normal	N	f _o
unbalanced fundamental	UF	f _o
harmonic-3	H-3	3 f _o
quasi-periodic 1/2	QP-1/2	$\approx f_0/2$
quasi-periodic 2	QP-2	$\approx 2 f_{o}$

The modes N, UF and H-3 are purely periodic while QP-1/2 and QP-2 have harmonic components whose frequencies are combinations of two basic frequencies that, in general, are incommensurable.

When varying a parameter in the circuit (voltage source amplitude, zero sequence capacitance, ...) a cascade of bifurcations may occur, with period doubling, and may lead to chaotic behaviour.

1548

The modes UF and QP-1/2, which are most likely to occur in practice, are now briefly described in order to show the zero sequence character of the ferroresonant oscillation.

Unbalanced Fundamental Mode

The unbalanced character of the UF mode in the three phases is related to a displacement of the neutral of the voltage source with respect to the neutral of the transformers.

Fig. 4 gives a representation of a typical evolution in the Clarke components state space : in the o component plane (fig. 4 above), the state variables ϕ_0 and u_n oscillate at the resonance frequency of the o component circuit (cfr fig 3.a) which is equal here to the frequency of the source ; in the $(\phi_{\alpha} - \phi_{\beta})$ plane (fig. 4 below), the evolution is nearly identical to that of the normal mode (the markers on the curves correspond to successive positive passages through zero of phase 1 of the voltage source.).

Quasi-periodic 1/2

Fig. 5 shows the evolution of the ϕ_0 and the ϕ_α Clarke flux components for a typical quasi-periodic 1/2 mode. The state variables u_n and ϕ_0 are almost sinusoidal at a frequency f_1 slightly lower than $f_0/2$: this slip can be visualized by the slow shift of the markers (positive passages through zero of phase 1 of the voltage source). The state variables ϕ_α and ϕ_β are almost sinusoidal at frequency f_0 with a slowly varying mean value at frequency $f_2 = f_0 - 2 f_1$. Fig. 6 shows the evolution during 10 periods of the source in the same state planes as above : in the o component plane ($\phi_0 - u_n$), the resonance at frequency f_1 can be seen ; in the ($\phi_\alpha - \phi_\beta$) plane, one has the source driven oscillation (solid line circles) with a slowly varying shift (circle of markers drawn for 36 periods).



Fig. 4 Case UF : state space representation



Fig. 5 Case QP-1/2 : time representation of ϕ_0 and ϕ_{α}



Fig. 6 Case OP-1/2 : state space representation

freq		u _n	φ _O	¢α	φβ
(Hz)		(kV)	(Wb)	(Wb)	(Wb)
1.35	$f_2 = f_0 - 2 f_1$	0.000	0.00	11.89	11.89
4.05	3 f ₂	0.003	0.24	0.00	0.00
21.62	f ₁ - 2 f ₂	0.000	0.00	0.17	0.17
24.32	f ₁	7.165	81.34	0.00	0.00
27.03	$f_1 + 2 f_2$	0.000	0.00	0.44	0.44
44.60	f _o - 4 f ₂	0.117	0.73	0.00	0.00
47.30	f _o - 2 f ₂	0.000	0.00	0.32	0.32
50.00	fo	0.000	0.00	20.81	20.81
52.70	$f_0 + 2 f_2$	0.132	0.70	0.00	0.00
72.97	3 f ₁	0.834	3.20	0.00	0.00
98.65	$2 f_0 - f_2$	0.000	0.00	0.38	0.38
101.35	$2 f_0 + f_2$	0.107	0.30	0.00	0.00
121.62	5 f ₁	0.094	0.22	0.00	0.00
124.32	$2 f_0 + f_1$	0.000	0.00	0.11	0.11

The amplitude of the main harmonic components is given in the next table : It may be seen that the harmonic components of the $(\phi_0 - u_n)$ variables do not appear for the $(\phi_\alpha - \phi_\beta)$ variables and conversely. The corresponding ϕ_α and ϕ_β components have the same amplitude and a phase shift of $\pi/2$. The same holds for the corresponding α and β components of the current. With exception of the f_0 component, the α (resp. β) components of the current are shifted in phase over $-\pi/2$ with regard to the corresponding α (resp. β) components of the flux. Therefore, they can be considered as energy sources at non-fundamental frequencies in the α and β circuits respectively.

5. Computation of the periodic oscillations

In order to compute periodic oscillations of the circuit of fig. 2, we use the harmonic balance method, which is a particular case of the Galerkin method : the flux in each nonlinear inductance (magnetizing branch) is represented by a limited Fourier series :

$$\phi(t) = \Phi_0 + \sum_{k \in K} \Phi_{k,c} \cos k\omega t + \Phi_{k,s} \sin k\omega t \qquad (9)$$

where k is an integer or a fraction. The corresponding Fourier coefficients for the current in this branch are given by integration from the magnetic characteristic $i(\phi)$ of this inductance :

$$I_{k,c} = \frac{2}{T} \int_{0}^{T} i(\phi(t)) \cos k\omega t \, dt$$
 (10)

and with similar expressions for the sine terms $I_{k,s}$ and for the DC component I_{o}

The harmonic balance method consists in introducing the limited Fourier series in the differential equations of the circuit and forcing to zero the contributions to each considered harmonic component. In this way, an algebraic system of nonlinear equations in the Fourier coefficients is obtained and may be solved by using a general purpose routine. A first attempt in this way is found in [7].

When a periodic oscillation is computed, its stability must be determined. A general method is to compute the characteristic exponents of the linearized variational equation of the initial system. But the method described in [3] is easier to use. Hereafter, we only give the result without demonstration.

Let us consider a set K' of harmonic components including the set K used for the computation of the periodic oscillation and

- even harmonic terms if K has none
- harmonic terms with frequencies that are multiples of half the fundamental frequency of the oscillation otherwise.

Using the harmonic balance method, the equations corresponding to the set K' may be written :

$$\mathbf{F}_{i}'(\tilde{\Phi}_{1},...,\tilde{\Phi}_{n'}) = 0$$
 $i=1,...,n'$ (11)

where the $\overline{\Phi}_j$ are written for the terms Φ_o , $\Phi_{k,c}$, $\Phi_{k,s}$. When computing periodic oscillations in function of the voltage source, the points where the relation

$$\det\left(\frac{\partial \mathbf{F}_{\mathbf{i}}}{\partial \tilde{\Phi}_{\mathbf{j}}}\right) = \mathbf{0} \tag{12}$$

is satisfied define the stability limits of the oscillations i.e. the points where bifurcations occur.

This property allows to compute directly the stability domains of the ferroresonant oscillations : adding (12) to the system of equations obtained by the harmonic balance method and considering, beside the Fourier coefficients, the voltage source as unknown, a new system of equations is obtained whose solutions give the Fourier coefficients and the source voltages corresponding to the stability limits of ferroresonant oscillations.

6. Approximation of quasi-periodic oscillations

The harmonic balance method assumes that the evolution is periodic and considers some a priori fixed harmonic components. Therefore, quasi-periodic oscillations can not be dealt with. A way to overcome this is to approximate the quasi-periodic oscillations with periodic ones. Let us consider again fig. 3. If the series impedances R_s are replaced in the three circuits with three resistances R_0 , R_α and R_β considered independent, the slow frequency f_2 is approximately proportional to $R_0 \cdot (R_\alpha \cdot R_\beta)^{1/2}$ (this is not true if R_0 tends to zero). If one of the resistances R_α or R_β is set to zero, the quasi-periodic oscillations become periodic : the flux ϕ_α (if R_α =0) or ϕ_β (if R_β =0) is directly related to the sinusoidal source, with an arbitrary additive constant.

Choosing $R_{\beta} = 0$, (5c) can be integrated. With initial condition zero, this yields :

$$\phi_{\beta}(t) = -\frac{U}{2\pi f_{o}} \sin 2\pi f_{o} t \tag{13}$$

With the same circuit parameters as those used to illustrate the QP-1/2 mode (paragr. 4), the amplitude of the main harmonic components of the periodic 1/2 oscillation are shown in the table given hereafter. It may be observed that the amplitude of the predominant harmonics are very close to those obtained for the QP-1/2 mode. The calculation of the currents shows the appearance of a DC component in the β circuit, whereas the DC component of the current is zero in the o and the α circuits.

freq		un	φ _O	¢α	фβ
(Hz)		(kV)	(Wb)	(Wb)	(Wb)
0.00		0.000	0.31	11.84	0.00
25.00	$f_1 = f_0/2$	7.482	82.65	0.53	0.00
50.00	f_o	0.153	0.85	20.77	21.04
75.00		0.848	3.17	0.08	0.00
100.00		0.136	0.38	0.35	0.00
125.00		0.067	0.15	0.11	0.00

Another way of approximating the quasi-periodic oscillations consists in applying the harmonic balance method directly to the original system of equations (without simplification), by taking, for the QP-1/2 mode, $K = \{0, 1/2, 1, 3/2, ... \}$. We have observed that numerical results are slightly less accurate with this method than with the first one.

7. Field case

From time to time, ferroresonance phenomena are observed in feeding networks of power plants auxiliaries or distribution networks within factories. The example described hereafter concerns the auxiliaries of a pumped storage power plant. Sometimes, when energizing the 6.6 kV cable feeder, a quasi-periodic 1/2 phenomenon was recorded with a beat period of 2.72 s (136 periods of the supply voltage).

Data received from the manufacturer permitted to determine the parameters of the VT : $R_s = 700 \Omega$, $R_p = 2.0 M\Omega$, $k_1 = 71.8 \ 10^{-6} A/Wb$, $k_5 = 2.58 \ 10^{-9} A/Wb^5$. The stability domains in the plane (U, Co) of the various ferroresonant modes were computed for the circuit without damping resistor. The results for the UF and QP-1/2 modes are shown on fig. 7. Big markers represent limit points obtained by time simulations, whereas small markers correspond to the direct calculation explained in paragr. 5 and 6. It may be seen that the limit obtained by approximating the quasiperiodic modes by periodic ones (dashed line) is very close to that obtained without this approximation (solid line) : a small discrepancy appears in the region where f_2 is not very small with respect to f_0 .

The zero sequence capacitance in the plant was unknown. Simulation of the QP-1/2 mode for the nominal voltage and various capacitances showed that the measured beat period is obtained for $C_0 = 300$ nF (point A on fig. 7). The computation of stability domains of the QP-1/2 mode taking into account damping resistors in the delta connection of the tertiary windings showed that, choosing $R_d = 0.5 M\Omega$ (related to the primary side of the V.T.), the risk of ferroresonance disappears (fig. 7 : dotted line).





Fig. 7 Stability domains of ferroresonant oscillations

8. Conclusion

A basic three-phase network with isolated neutral has been considered. In such a situation, the interaction between the saturable voltage transformers and the capacitances of the circuit may lead to three-phase ferroresonance phenomena. A comprehensive physical interpretation is found by using the Clarke transformation : they correspond to resonant oscillations of the o component (zero sequence) circuit ; these oscillations are sustained due to the coupling with the circuits of the α and β components introduced by the nonlinear reactors.

The harmonic balance method, completed by an equation expressing that the determinant of a Jacobian is equal to zero, has been used here. When compared with time domain simulations, this method appears particularly appropriate for a rapid determination of the stability domains of the different ferroresonant oscillation modes in the plane (source voltage U - zero sequence capacitance C_0) and to design a damping circuit.

A slight modification of the system parameters allows a study of the quasi-periodic oscillations by forcing a periodic behaviour.

Finally, a field case has been presented.

The model of the voltage transformers used in this paper is rather crude. A more detailed representation will be included in the near future in order to get quantitatively more reliable results.

References

- [1] N. Germay, S. Mastero, J. Vroman, "Review of Ferro-Resonance Phenomena in High Voltage Power Systems and Presentation of a Voltage Transformer Model for predetermining them," CIGRE 1974, report 33-18.
- [2] I.A. Wright, "Three-phase subharmonic Oscillations in Symmetrical Power Systems," IEEE Trans. on PAS Vol 90, May-June 1971, pp 1295-1304
- [3] N. Janssens, "Calcul des zones d'existence des régimes ferrorésonants pour un circuit monophasé," IEEE Canadian Communications & Power Conf., Montreal 18-20 Oct 1978 Cat N°78 CH 1373-0 REG 7, pp 328-331
- [4] N. Janssens, A. Even, H. Denoel, P-A. Monfils, "Determination of the Risk of Ferroresonance in High Voltage Networks. Experimental Verification on a 245 kV Voltage Transformer," Sixth International Symposium on High Voltage Engineering. New Orleans, Aug 28 - Sep 1 1989.
- [5] H.A. Peterson, Transients in Power Systems, New York: Wiley, 1951, pp 265-279
- [6] C. Bergmann, "Grundlegende Untersuchungen über Kippschwingungen in Drehstromnetzen," ETZ-A Bd 88 (1967) H.12, pp 292-298
- [7] A.J. Germond, "Computation of Ferroresonant Overvoltages in Actual Power Systems by Galerkin's Method," IEEE PICA 1975 Conf., New Orleans, pp 127-135

Biographies

Noël Janssens was born in 1948. He is Electrical Engineer from the University of Louvain in Belgium (1971) and obtained the Ph.D. degree in 1981 (modelling of magnetic hysteresis and study of ferroresonance). From 1981 to 1983, he worked at ACEC (Charleroi) as head for R & D in the On Load Tap Changer department. From 1978 to 1981 and since 1984 he is with Laborelec, where his main fields of interest are the modelling, simulation and control of Power Systems. He is also teaching at the University of Louvain (Louvain-la-Neuve) in the Electrical Engineering department.

Thierry Van Craenenbroeck was born in 1966. He graduated in 1989 as Electrical Engineer from the Katholieke Universiteit Leuven. From 1990 to 1992 he worked as lecturer at the Anton-de-Kom University of Surinam. From 1992 on, he is working at K.U.Leuven towards a Ph.D. degree on three-phase ferroresonance in distribution networks.

Daniel Van Dommelen (SM '78) is Electrical Engineer from the K.U.Leuven in Belgium, has an M.Sc. in Electrical Engineering (U. Wisc.), and a PhD from the K.U.Leuven. Since 1977 he is full professor at this university and head of the laboratory for Power Systems, High Voltage and Electroheat. He is author of a book and of numerous publications in both national and international journals. He is chairman and Belgian representative in the ERE Committee of the UIE. He has been chairman of the IEEE Benelux Section, and is a member of IEEE PES, CIGRE, SEE and national electrical engineering societies.

Félicien Van De Meulebroeke (SM'89) was born in 1940. He received the Electrical Engineer degree (Electronics) from the Catholic University of Leuven (K.U.Leuven), Belgium, in 1963. Since 1964, he has been with Laborelec, the Belgian Laboratory of the Electricity Industry, successively Chief Engineer in charge of turbogenerators automation and system dynamics, head of the department "System dynamics and protections" (1987), head of the Electrical Division of Laborelec (1988). Since 1994, he is with Electrabel, where he is director of the HV Transmission Network exploitation.

Discussion

VITALY FAYBISOVICH, FPS Consulting, Los Angeles, CA:

This paper describes using of Clarke components to write the equations of a representative three-phase circuit under a canonical form and give a simple physical interpretation of the three-phase ferroresonance phenomenon. The solutions and their domains of stability are computed by the harmonic balance method.

The authors' comments to the following questions would be greatly appreciated:

1) For using harmonic balance method to calculate of the stability domains the number of anticipated harmonics and their frequencies should be chosen in advance. According paragraph 4 of this paper five different modes of operation with different harmonics content can be expected. How the authors chose anticipated harmonics to built stability domains for different modes of oscillations?

2) According paragraph 5 of the discussed paper the harmonic balance method permits to compute the stability domains in coordinates of Fourier coefficients (harmonic amplitudes) and source voltage. The stability domains at Fig. 7 are presented for voltage source - capacitance coordinates. How the harmonic balance method was used to receive these stability domains?

3) From Fig. 7 it is evidently that stability domains for different modes of ferroresonance oscillations are overlapped. This means that for some combinations of voltage source amplitude and circuit capacitance the UP or QP 1/2 modes of oscillation can be expected. Is it possible to predict the anticipated mode of oscillation in such area and can harmonic balance method be used for such study?

Manuscript received August 18, 1995.

N. Janssens, Th. Van Craenenbroeck, D. Van Dommelen, F. Van De Meulebroeke.:

The authors are grateful to the discusser for his interest in the paper.

Question 1 : In order to get the best computational efficiency, the set of harmonic components, used to compute an oscillation mode, is particular to this mode. The predominant components can be identified using a FFT on time simulation results. For instance, the unbalanced fundamental mode was represented by the components 1, 3, 5, 7 and 9 whereas the quasi-periodic 1/2 mode was represented by the components 0, 1/2, 1, 3/2, 2 and 5/2.

Question 2 : The combination of the harmonic balance equations with the stability criterion gives, for a fixed set of circuit parameters (voltage source frequency, capacitance, resistances, magnetic characteristic, damping resistance), a boundary of the voltage source amplitude interval (and the corresponding harmonic components of the fluxes) where an oscillation mode is stable. Successive calculations performed with different values of the capacitance give the boundary line of the stability domain in the plane (voltage source amplitude - capacitance). The solution of a calculation isused as initial point for the next one.

Question 3 : The aim of the harmonic balance method is to compute directly periodic oscillations. In this way, the calculation of the transient evolutions is avoided. This method does not allow to determine the final oscillation reached starting from given initial conditions. The domain of attraction of each stable steady state solution may only be determined by numerous time simulations. However, the limits between the basins of attractions are fractals and their computation is cumbersome. Some examples are given in the book "Nonlinear Oscillations in Physical Systems" by C. Hayashi (Mc Graw Hill, 1964), chapter 10.

From a practical point of view, due to the uncertainty about the initial conditions and about the various events that can affect the system, the best way to avoid ferroresonance is to be sure that terroresonance cannot occur, whatever the initial conditions. Therefore, the working point of the system under study must be outside the stability domains of all the abnormal oscillation modes.

Manuscript received October 17, 1995.