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Analysing Time-Varying Power System Harmonics Using Wavelet Transform

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***Abstract* - This paper presents the possibilities offered by the dyadic-orthonormal wavelet transform used in the multiresolution analysis of voltage- and current-signals. This transform proves to have some advantages over the classical FFT-based algorithms, when used in electric power quality assessment and the analysis of waveforms. Practical examples using waveforms generated by energy saving lighting equipment, remote-control signals and an adjustable speed drive, are presented.**

I. INTRODUCTION

During the last years there is a growing interest in the quality of electric power. This can be explained partly by the widespread use of nonlinear fast-switching electronic equipment in the industrial environment. These devices are sensitive to distortions in their supply-voltage, but they are also an important contributor to the deterioration of the quality, since they often inject highly distorted currents in the net. To evaluate these phenomena, appropriate measuring techniques have to be used [1].

An important factor determining this electric power quality is the harmonic pollution present in the voltage and current waveforms [2]. In the classical measurement algorithms, the harmonic content of these signals is determined by the application of a Fast Fourier Transform (FFT), a finite version of the Discrete Fourier Transform:

$$F[n]=\sum_{k=0}^{N-1} f[k]\cdot e^{-\left(\frac{j2\pi kn}{N}\right)} \qquad (1)$$

However, the application of this technique will only offer correct and accurate results if some implicit assumptions about the sampled signal are met. Nowadays, it becomes harder to assume that the waveforms show this well-described behavior: loads become more and more dynamic and thus the generated harmonics become more and more complex and vary faster in time. Therefore the need for new methods arises.

In this paper we present a new mathematical method that can be used in cases where an FFT comes short, namely the dyadic-orthonormal wavelet transform, used in the multiresolution analysis (MRA) of waveforms. At first, the pitfalls of the FFT are discussed, followed by an introduction to wavelet analysis and a discussion on the implementation of this technique. Finally the use and the advantages of the MRA will be demonstrated by three examples in which an FFT is not suitable or doesn't reveal all the information.

II. FFT-BASED ANALYSIS

A. Conditions

The Fast Fourier Transform has become a popular method to analyse signals containing more than one frequency. In electrical power engineering this is mostly the fundamental frequency of 50 or 60 Hz plus harmonics, having a frequency which is an integer multiple of this fundamental frequency. In extreme cases subharmonics with lower frequencies are also possible. Interharmonics, who lie in between, may occur as they are sometimes used for remote-control purposes.

For practical reasons, the continuous signal has to be sampled. From the mathematical derivation of the FFT, it follows that the signal is proposed to be infinitely repeating in the present and in the future. This causes three conditions to be met:

(1) The Nyquist criterion says that the sampling frequency has to be greater than the double of the highest occuring frequency.

(2) The waveform must be assumed stationary and periodic.

(3) For the correct determination of the real frequencies, an integer number of the periods of all the components must be included in the window.

B. Error-Causing Effects

If the signal or its set of samples doesn't meet these conditions, inaccuracies can be found in the spectrum generated by the FFT.

The best known effect that can falsify the results (the spectrum) is probably "aliasing". This means that spectral components with a frequency above the Nyquist limit influence the amplitude of frequencies below that limit. A higher sampling frequency or a low-pass filter can minimize this effect.

Another effect is called "spectral leakage" [3]: this occurs when the window doesn't contain an integer number of periods. In that case the energy present in a certain component will spread out and make the peak in the spectrum less sharp and lower; a mutual interference between the spectral lines is produced. To avoid this kind of incorrectness, a phase locked loop controlling the sampling process should be applied and/or a weighting window should be used to adapt the samples.

If the signal contains components at non-integer multiples of the fundamental frequency, the so-called "picket-fence effect" can be found in the spectrum. This can be seen as a special kind of leakage at certain frequencies. For instance these frequencies can exist in transient phenomena, ferroresonance or time-varying harmonics, ... [4].

C. Short-Time Fourier Transform

To solve the errors caused by transient phenomena, the Short Time Fourier Transform was developed (STFT, sometimes refered to as the Gabor Transform) [5], originally applied in sound analysis. The main difference between this technique and the FFT, is that the FFT uses large consecutive windows in time to sample the signal. The Windowed Discrete Fourier Transform (WDFT; the discrete version of the STFT) uses a window that translates in time. In this sliding window the transform is calculated at different moments in time, thus creating the effect of a set of band-pass filters, with every filter having the same width Δf, determined by the sampling rate and the window itself. The disadvantage of this algorithm is the trade-off that has to be made between the length of the window (typically a cycle of the fundamental frequency or less; this determines the resolution in time) and the frequency resolution Δf in the spectrum. It's obvious that when a one-cycle window is used, the frequency resolution equals the fundamental frequency. In such cases, a lot of information is lost in the spectrum.

Now the signals can be represented in two-dimensional grids (fig. 1). The divisions on the horizontal axis represent the width in time of each window of the WDFT. The divisions on the vertical axis represent the band-pass filters with their frequency-extent. For each rectangle a value is generated that states the amount of signal within that specific frequency interval and during that specific time-window.

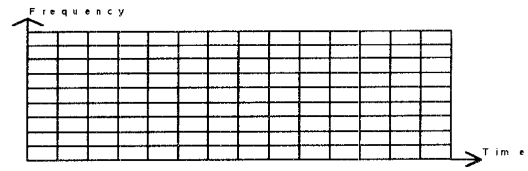


Figure 1: Classical time-frequency plane

However, usually the 41st and the 43rd harmonic in the spectrum don't have to be determined with the same accuracy as the 3rd and 5th harmonic. In such cases one would rather want a kind of multiresolution frequency analysis in which the filter-width Δf isn't a constant anymore. Also the following remark can be made: a lot of the low-frequency phenomena have a relatively long duration, whereas high-frequency phenomena last only relatively short. Thus in fact, multiple time-scales should be used too. A preferred time-frequency diagram could be the one in fig. 2.

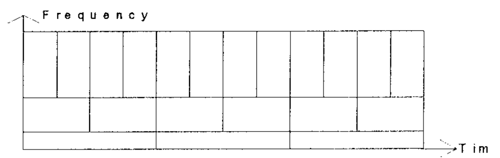


Figure 2: Alternative time-frequency plane

It's obvious that Fourier based methods don't offer such multiresolution techniques. As shown further on, wavelet analysis does.

III. WAVELET ANALYSIS

A. Fundamentals

The Fourier transform decomposes the signal in an orthonormal basis containing sin- and cos-functions, who are infinitely extending in time. On the other hand, the Wavelet Transform, proposed here, decomposes the waveform in a basis of signals who are all finite in time and in frequency content. These signals are also orthonormal and thus the decomposition will be unique.

The Continuous Wavelet Transform (CWT) correlates the studied signal with a signal $\psi(t)$, called a "wavelet", being a real, small and smooth wave with a finite duration in time and a finite frequency content:

$$S(b,a) = a^{-1/2} \cdot \int_{-\infty}^{\infty} \psi_{a,b}(t) \cdot f(t)\, dt \qquad (2)$$

with $\psi_{a,b}(t)$ a function derived from the so-called "mother wavelet" $\psi(t)$:

$$\psi_{a,b}(t) = a^{-1/2} \cdot \psi\left(\frac{t-b}{a}\right) \tag{3}$$

This function is scaled (dilated) by the parameter a, thus defining a frequency resolution and translated in the time domain by the parameter b.

In theory the number of possibilities for this mother wavelet is infinite. Any function that is finite in time and frequency is suitable. Numbers of authors have developed their mother wavelet with special properties that make it suitable for application in different fields. Examples are the Coiflet, the Symmlet, the Morlet, the Haar-, the Daubechies- and the Meyer-Wavelet [6].

To use this transform with sampled data, the Discrete Wavelet Transform (DWT) is used:

$$S(a,b) = a^{-1/2} \cdot \sum_k f[k] \cdot \psi_{m,n}[k] \tag{4}$$

This is a finite sum because of the finite length of the function $\psi_{m,n}[k]$, that is derived from the mother wavelet by the following formula:

$$\psi_{m,n}[k] = a_0^{-m/2} \cdot \psi\left[\frac{k - nb_0}{a_0^m}\right] \tag{5}$$

If $a_0 = 2$ and $b_0 = a_0{}^m = 2^m$ is chosen, then this family of wavelets forms an orthonormal basis in which a function can be decomposed in a unique way. Because of the choice for the scaling parameter, which is a power of two, this decomposition is also called "dyadic". By increasing the parameter *m*, the length in time of the wavelet decreases logarithmically. Also the frequency content of this signal increases logarithmically. The parameter *n* translates the function in time. Thus the above mentioned special multiresolution technique of fig. 2 can be implemented based on these wavelets.

B. Implementation

The previous discussion suggests the construction of a filter bank with logarithmic widening filters. This can be implemented in an efficient way in a so-called pyramidal algorithm. This means that based on the set of wavelets and their mathematically closely related set of scaling functions $\phi(t)$ consecutive sets of high pass filters (HPF) and their complementary low-pass filters (LPF) can be derived (fig.3). (These scaling functions form the basis for the orthogonal complement of the space described by the wavelet basis). This is done by an elegant and fast calculation.

In signal processing, these complementary filters are called quadrature mirror filters (QMF). The HPF strips off the high-frequent "details" in the signal, whereas the LPF produces the remaining coarser signal. This last signal has now a diminished bandwidth and can thus be downsampled. Then it will pass through the next set of QMF to produce details at another frequency ("scale" or "dyad"). In this way the multiresolution analysis (MRA), as it was first described by Mallat, is established [6].

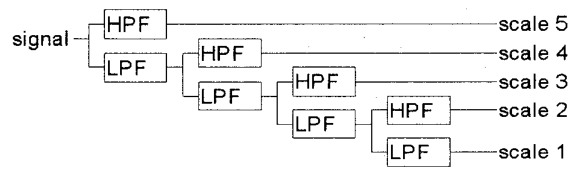
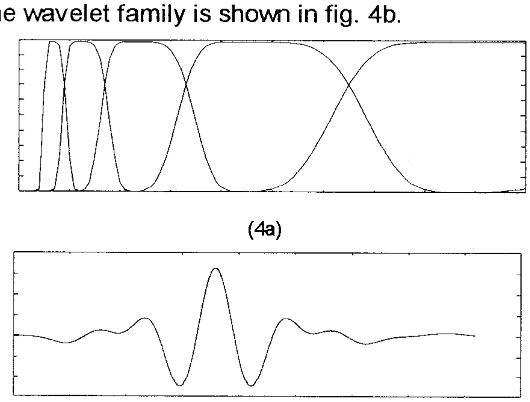


Figure 3: Pyramidal algorithm with QMF-bank

C. Choice of the Mother Wavelet

As mentioned before, there is an enormous degree of freedom, namely the choice of the wavelet basis, determined by the mother wavelet, in which the decomposition takes place. Each of these mother wavelets has special properties that makes it suitable for a special kind of signals. For instance, the Daubechies wavelets are often used in image compression and, recently, to detect electromagnetic transients produced by power system faults or switching [7].

In this research we want to use the wavelets to analyse time-varying power system harmonics. Thus a flat band-pass filter characteristic and a cut-off as sharp as possible are required. The Meyer-Wavelet has these properties. A part of the filter bank created with this Meyer-Wavelet is shown in fig. 4a, while a member of the wavelet family is shown in fig. 4b.



(4a)

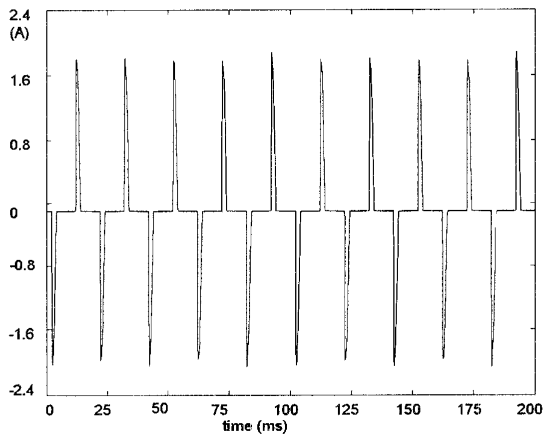
(4b)

Figure 4: (a) Meyer-Wavelet filter bank in the frequency domain; (b) Member of the Meyer-Wavelet family in the time domain

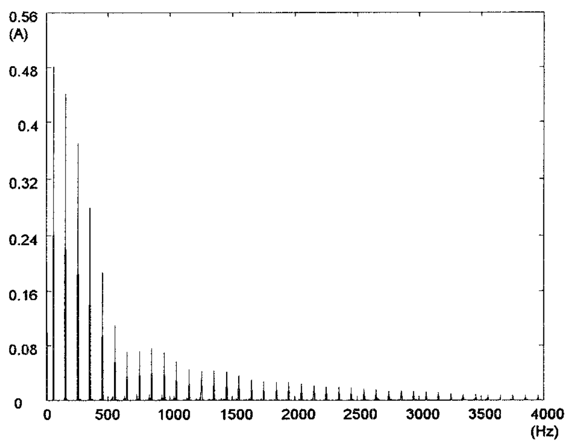
IV. PRACTICAL EXAMPLES

A. Example 1: Current Drawn by Energy-Saving Lighting Equipment

As a first example the current drawn by a small "energy saving" lamp of 20 W is used. In fig. 5a, which is a plot of 4096 samples over exactly 10 periods of the current (fundamental frequency of 50 Hz) it can be seen that every current peak has a different height, and thus periodicity is lost. This is explained by the stochastic character of the ignition in the lamp combined with a finite resolution in the sampling process. The FFT of the signal shows in consequence a spreading of the energy (fig. 5b).



(5a)



(5b)

Figure 5: (a) Current signal; (b) FFT (example 1)

The MRA is given in fig. 6. The first and second scale are not given since no components lying in these very small frequency windows were present. On scale 3, the wave modulating the amplitudes shows up. The signal at the fundamental frequency appears on scales 4 and 5. This occurence on two scales can be explained by the non-ideal filter characteristics (see also fig. 4a): due to the overlap of the filters, signals at certain frequencies can be found on more than one scale. Most of this 50 Hz signal is situated on the fourth scale; on the fifth scale the minor remaining part is mixed with a bit of the third harmonic that is dominant on the sixth scale.

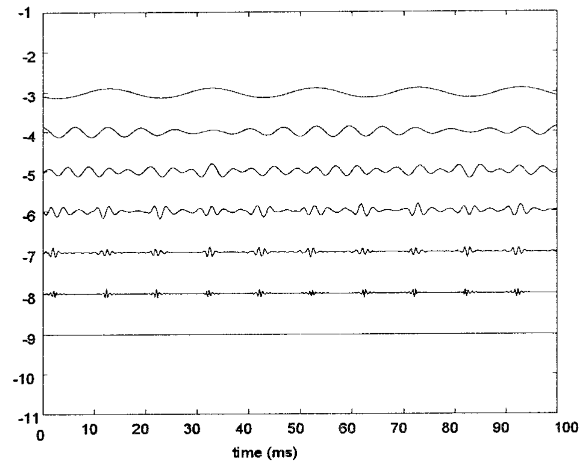


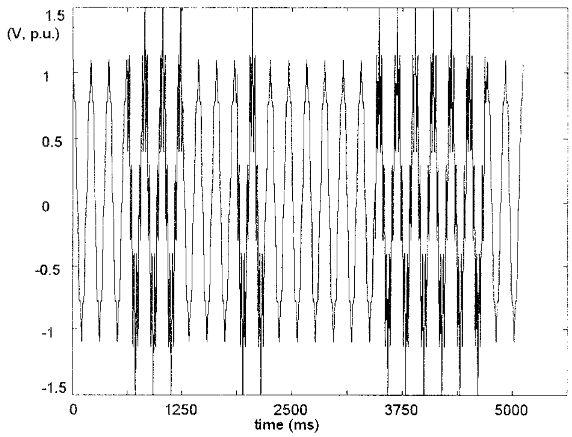
Figure 6: MRA (example 1)

The attention has to be drawn on the non-constant amplitude of the signal at this sixth scale. Usually when the term "harmonics" is used, we are inclined to think of signals with a constant amplitude, like in the Fourier series. With this kind of analysis, a time-varying characteristic shows up, which is overseen by the classical FFT-based analysis. In this context, the term "harmonic order" isn't, perhaps, the right one to use. Maybe a term like "(Meyer-)Wavelet order" is better.

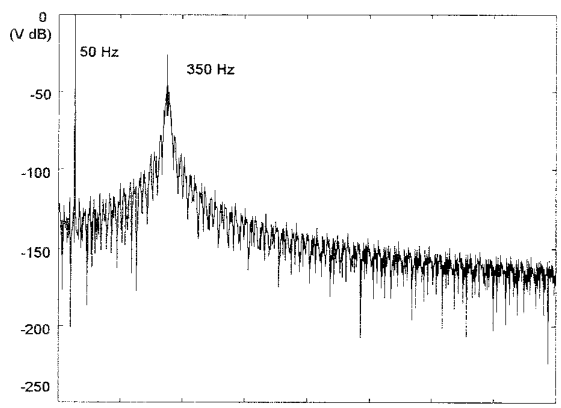
From the seventh scale on, the frequency components, present in the sharp edge of the rising side of the current show up. When the scale is higher (plotted lower on the figure), the signal becomes more situated in time and the frequency window gets broader. On these higher scales signals of higher harmonic order are taken together on the same scale. That offers the possibility to see the lower order signals with a sure distinction, more precisely on the lower scales, where the frequency window is small, but the time window is bigger. In this example it is shown that the signal is decomposed in the time-frequency plane of fig. 2.

C. Example 2: Remote-Control Signals

The next example shows the simulation of 8192 samples of a slightly distorted voltage, with superimposed a few pulses as used by some types of remote-control signals (fig. 9a). In this example the pulses consist of pulsed waves with the same frequency as the distorting component (350 Hz). In the FFT (given here with a logarithmic scale) the sharp edged fundamental is found on the left, but the pulsation at 350 Hz causes a strong energy leak that affects the whole spectrum (fig. 9b).



(9a)



(9b)

Figure 9: (a) Voltage signal; (b) FFT with a logarithmic voltage scale (dB) (example 2)

In the MRA (fig. 10), the fundamental is completely situated on the fifth scale. On the eighth scale, the distortion plus the control signals are filtered out. On the other scales the high frequency components including the sharp edges at the rising and decaying of the pulses can be found. With the help of these scales, the exact length and occurence in time of the pulses can be determined.

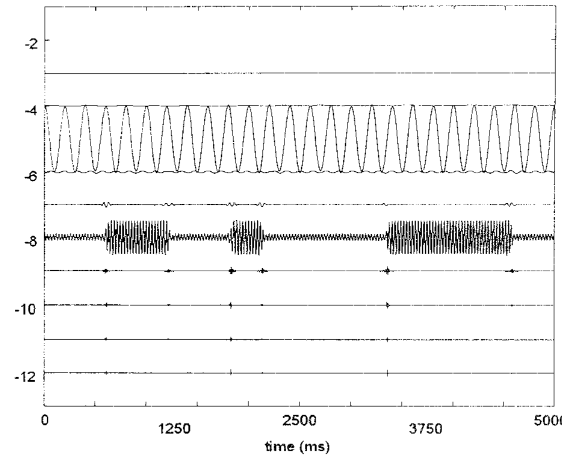
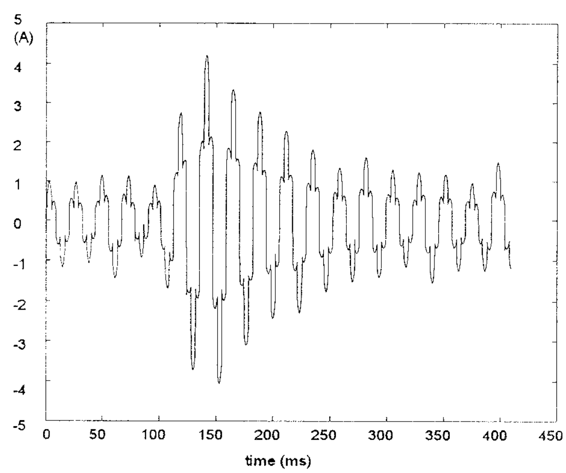


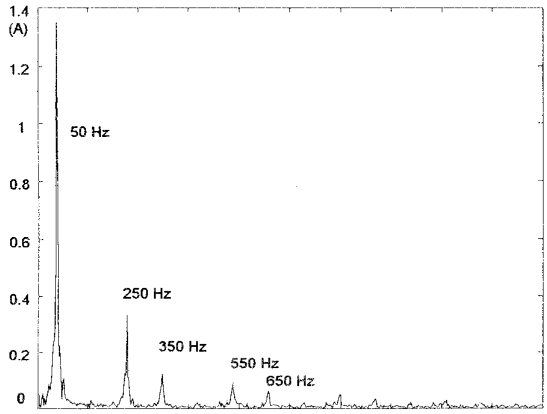
Figure 10: MRA (example 2)

D. Example 3: An Adjustable Speed Drive

The final example shows 4096 samples of the measured current drawn in one phase of a fully controlled three-phase rectifier bridge feeding a DC-motor. The bridge is connected to the net by a Δ-Y-transformer. The setup is controlled by a fast current- and a fast speed-controller. Even in "stationary" conditions the signal is far from stationary because of the high dynamics in the controller. In this particular example a small speed change was commanded to the speed regulator. This results in a short rise of the current to accelerate the motor, after which the situation becomes "stationary" again (fig. 11a).



(11a)



(11b)
Figure 11: (a) Current signal; (b) FFT (example 3)

The FFT of the signal (fig. 11b) shows that there is a dominating fundamental at 50 Hz, and "harmonics" of order 5, 7, 11, 13, ... Since the peaks are far from sharp the amplitudes are unreliable. On the other hand, the MRA (fig. 12) does show the rise in fundamental current at scale 4 and 5. At scale 6, it can be seen that there is no signal of harmonic order 3 (because of the transformer Δ-winding). At scales 7 and 8, the relatively smaller rise of the harmonic current can be noticed. The small peaks at the other scales are the commutations within the bridge, that hence can clearly be determined. In this example, the MRA provides more information than the FFT.

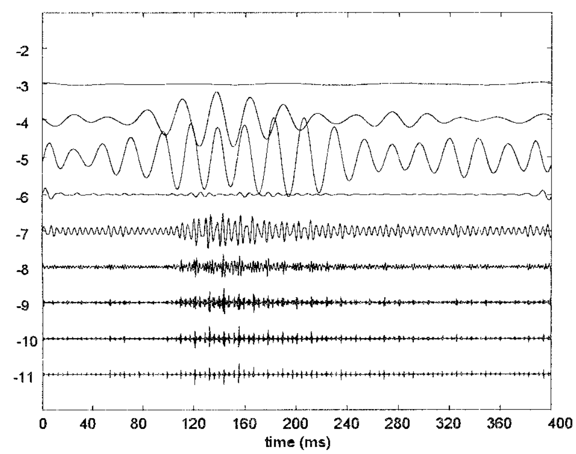


Figure 12: (a) MRA (example 3)

V. PRACTICAL USE OF THE MRA

These three examples try to prove that using the MRA more information and usually more relevant one than from applying an FFT can be obtained. To perform the calculations, no more operations had to be done than for an FFT. In the mathematical literature, even faster algorithms can be found [6]. With the recent evolutions in the field of the DSP-technology, the implementation of algorithms performing a wavelet-based decomposition, becomes possible. Future applications of this method can be:

- Analysing the behavior of devices, apparatus and systems based on simulations or measurements: the previous examples illustrate the possibilities that can be achieved.
- The monitoring of the power quality: it may be clear that not only harmonics can be detected, but also the moment of their appearance can be determined, including their transient behavior.
- Identification of phenomena appearing in energy supply systems [5,7].
- Remote-control systems: as in the example, a new method to intercept the remote-control signals in distorted conditions, can be developed.

VI. CONCLUSIONS

A new powerful analysing method, namely the wavelet-based multiresolution analysis, for current- and voltage-signals with time-varying power system harmonics has been proposed. This method might give better results in situations where the classical FFT-based analysing methods come short or give incorrect results. The Meyer-Wavelet has been suggested to be used in these situations. The practical implementation has been discussed. The advantages of this method are shown in three practical examples.

REFERENCES

[1] A.. Domijan, G.T. Heydt, A.P.S. Meliopoulos, S.S. Venkata, S. West, "Directions of Research on Electric Power Quality", IEEE Transactions on Power Delivery, Vol. 8, No. 1, pp. 429-436, January 1993

[2] J. Arrillaga, D.A. Bradley, P.S. Bodger, *Power System Harmonics*, Wiley Chicester, 1985

[3] K. Mikolajuk, Z. Staroszczyk, "Measurement Aspects of the Voltage Distortion Compensation", European Transactions on Electrical Power Engineering, Vol. 4, No. 5, Sept./Oct. 1994

[4] C.S. Moo, Y.N. Chang, P.P. Mok, "A Digital Measurement Scheme for Time-Varying Transient Harmonics", IEEE Transactions on Power Delivery, Vol. 10, No. 2, pp. 588-594, April 1995

[5] D.C. Robertson, O.I. Camps, J.S. Mayer, W.B. Gish, "Wavelets and Electromagnetic Power System Transients", 95 SM 391-3 PWRD, IEEE/PES Summer Meeting 1995

[6] Charles K. Chui, *An Introduction to Wavelets*, Academic press Boston, 1992

[7] S. Santoso, E.J. Powers, W.M. Grady, P. Hofmann, "Power Quality Assessment via Wavelet Transform Analysis", 95 SM 371-5 PWRD, IEEE/PES Summer Meeting 1995