Eddy-Current Calculation of Statistically distributed Material

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Abstract – **In this paper the calculation of eddy current loss of a statistically distributed material will be presented. For loss calculation two FE- and one semi-analytical formulation will be compared. The distribution of the material which is only known by some global parameters will be generated by a Monte Carlo approach, so that a large number of calculations describe one system state. The results will be evaluated by means of probability theory.**

I. Introduction

One of the requirements for the application of the FEM to a problem is the exact knowledge of the geometric dimensions and positions of all relevant parts. Sometimes this knowledge cannot be attained for the whole material distribution but only for some global parameters e.g. fill density.

An example for this is the partially molten charge of an induction furnace. These furnaces are as a rule charged with scrap that exists in different piece-goods-qualities from steel wool to coarse scrap pieces. Here the position and shape of all scrap particles is unknown. Even if these informations could be determined, the calculation costs would be very expensive. In this paper we propose another proceeding.

Below follows a description of these problems by a repeated generation of the local geometry that satisfies the global parameter volume fill density f, which is calculated from the mass m, the volume V_{Fill} and the mass density ρ by

$$f = \frac{m}{\rho\, V_{Fill}} \quad . \tag{1}$$

The material distribution will be produced by a random generator and finally calculated by one of the following methods.

I. Calculation of the induced loss

Caused by the quantity of necessary calculations for a statistical evaluation an efficient computation method for the field problem is necessary.
Below three calculation methods will be discussed:

- FE approach with the $\vec{A}$ integrodifferential formulation
- FE approach with the $\vec{A}V$ formulation
- Semi-analytical approach

A. FE approach with the $\vec{A}$ integrodifferential formulation

The integrodifferential formulation [1] eliminates the grad V term by using the following relation for the total current I.

$$I = -a\,\sigma \mathrm{grad}\, V - j\omega\sigma \iint A\, dS \tag{2}$$

Here a represents the area of the conductor cross section S. Linking the eddy current diffusion equation and (2) one obtains the matrix equation form

$$\left\{[S] + j\omega\sigma[T] - j\frac{\omega\sigma}{a}[T][C]\right\}[A] = [T][G] \tag{3}$$

with the well known matrices $[S]$ and $[T]$. Here $[C]$ is Konrad's integration matrix [1] and [G] the matrix of the global currents. The main disadvantage of this method is the high number of matrix entries and with this the high calculation time. Therefore the integrodifferential approach is only used to calculate conductor cross sections with just a few elements, which will reduce the above disadvantage.

B. FE approach with the $\vec{A}V$ formulation

The second formulation uses additional equations for the total current in each conductor (2). The term grad V can now be treated as unknown with the eddy current equation

$$\left\{[S] + j\omega\sigma[T]\right\}[A] + \sigma[T][\mathrm{grad}\, V] = 0 \tag{4}$$

The whole matrix then can be symmetrized [2].

C. Semi-analytical

In cases where the dimension of the cross section of a long conductor is small compared to the skin depth δ the secondary reaction of the induced current densities may be neglected. Then the loss power per unit length P_L in that conductor is given by [3]

$$P_L = \frac{4\,H_0^2}{\delta^4\,\sigma} \iint x^2\, dx\, dy = \frac{4\,H_0^2}{\delta^4\,\sigma}\, J_x \tag{5}$$

where σ is the conductivity, H_0 the field value, here in the y-direction, and J_x the moment of inertia around the x-axis (fig. 1).

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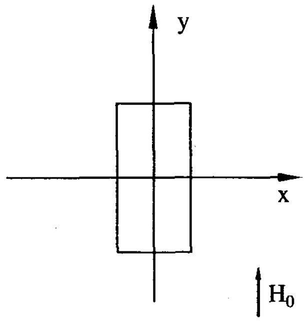
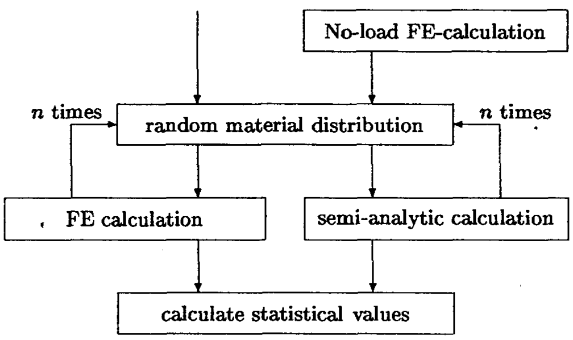


Fig. 1: Long conductor in a transverse field

II. Evaluating the results

In the following we consider a system state to be a volume filled with a conductive material of small particles with a certain volume fill density f where the exact position of a particle is unknown. This state can be achieved by a various number of particle distributions in the specified volume that satisfies the global parameter f (sample space). To describe the effect of a distributed material a large number, typically $n = 100$, of different independent distributions (trials) of the same sample space is generated by a random generator.



In the case of the semi-analytical approach the field value is obtained once by a FE analysis for the no load case without statistical material. The following analytical computations are very fast. In the other case after each generation of a new material distribution the field problem has to be solved by a FE calculation.

After solving the field problem for each trial one collects the desired results, e.g. field values, loss or efficiency. This gives a probability distribution function $P(X)$ of the results from which we can derive the density function by differentiation. As in many technical processes these independent trials result in a normal distribution. With the expected value $\bar{x}$, the variance σ_P and a confidence level c we obtain the confidence interval

$$\left[\bar{x} - c\frac{\sigma_P}{\sqrt{n}}\,;\,\bar{x} + c\frac{\sigma_P}{\sqrt{n}}\right] . \qquad (6)$$

III. Examples

The following FE calculation were carried out with second order triangular elements and particles that consist of two triangles. Each particle represents a single unshorted conductor of homogeneous material with quadratic shape.

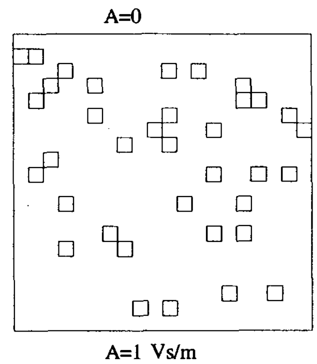


Fig. 2: Region with homogeneous field and some conductive particles

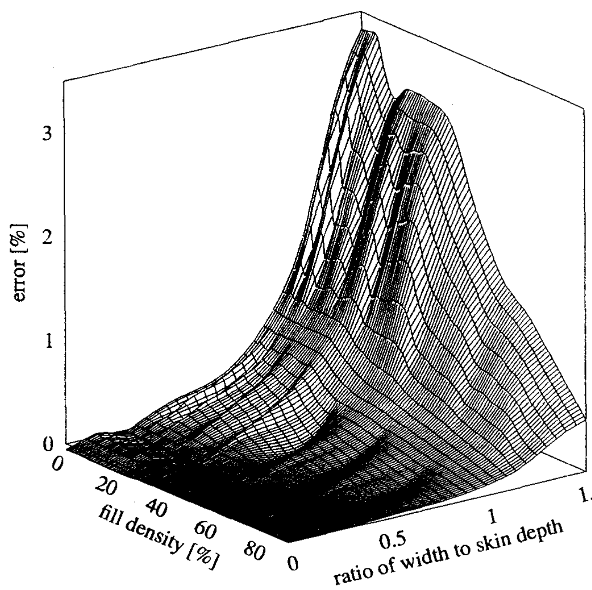


Fig. 3: Comparison of FE- and semi-analytical calculation

A. Particles in a homogeneous field

According to fig. 2 some particles were exposed to a homogeneous field realized by a vector potential difference between the lower and upper edges. 800 calculations for each the integrodifferential and for the semi-analytical formulations each have been carried out. The results of the two methods agree very well even for high ratios of perimeter to skin depth (fig. 3).

Comparing the two FE calculations the integrodifferential approach has a lower matrix dimension with only few additional matrix entries because of the discretisation of one conductor by only two triangular elements. Therefore it is a little faster (4%) than the $\vec{A}V$ formulation.

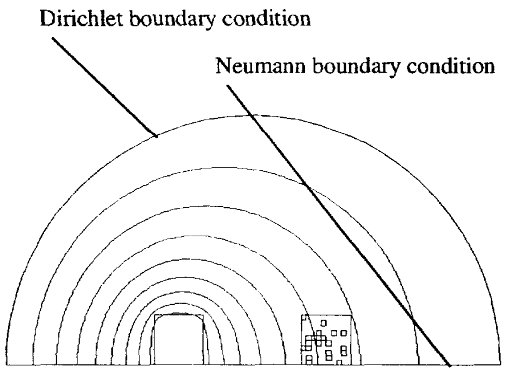


Fig. 4: Long conductor with equipotential lines

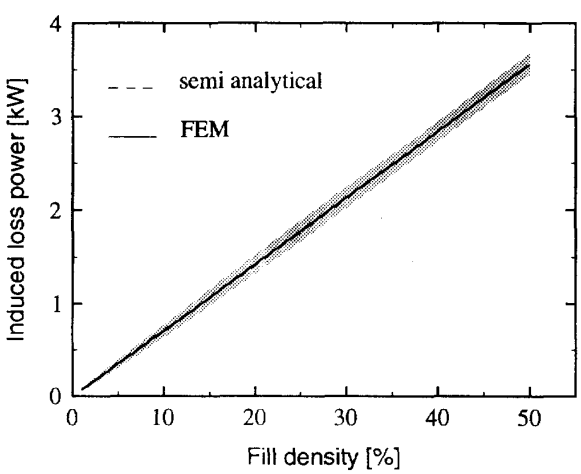


Fig. 5: Comparison of FE- and semi-analytical calculation

B. Eddy current field of a long line

Fig. 4 shows the geometry and equipotential lines for a long line with rectangular shape with an area of small material particles on the right. The ratio of particle perimeter to skin depth was 0.09. Here the FE calculations have been carried out by the $\vec{A}V$ formulation. The integrodifferential approach is significantly slower because the number of non-zero matrix elements has increased. Both 700 FE- and semi-analytical calculations have been made to compare the two approaches. From fig. 5 can be seen that the expected values show very good agreements and even the confidence intervals (for low fill densities about 8% of the mean value, for higher densities about 3%) are comparable.

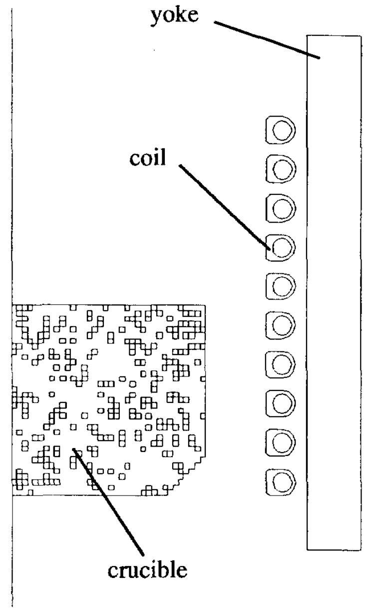


Fig. 6: Part of the axisymmetric FE model of an induction furnace

To estimate the computational error the variances of the over all loss in the model for second and third order elements were compared. The differences for the mean values and for the variances were some hundredth of a per cent of the mean value i.e. about two orders below the values of the material's distribution effect.

C. Eddy current field of an induction furnace

Induction furnaces are nowadays calculated with FE programs. Here only those states can be considered, where the crucible charge is homogeneous, e.g. totally liquid. For these states the calculations of the electromagnetical, fluid mechanical and temperature-fields are well known [4]. These homogeneous states only take little time of the operating time of the furnace. Mostly it is filled with an inhomogeneous mixture of liquid metal and solid metal particles. Those conditions can be described by some global parameters such as mass and filling height. The local distribution is unknown.

For the calculation of the induced power loss in a furnace a time-harmonic axisymmetric model of the furnace

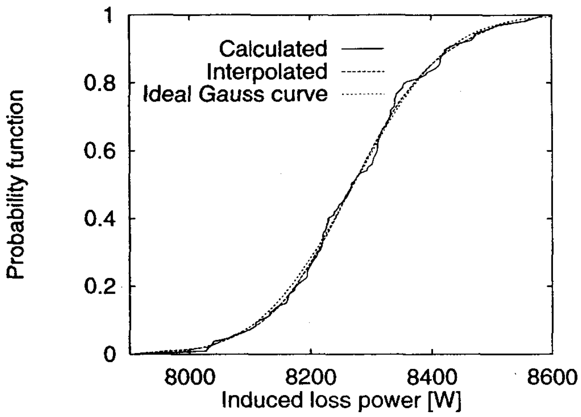


Fig. 7: Probability distribution of 100 trials for f=20%

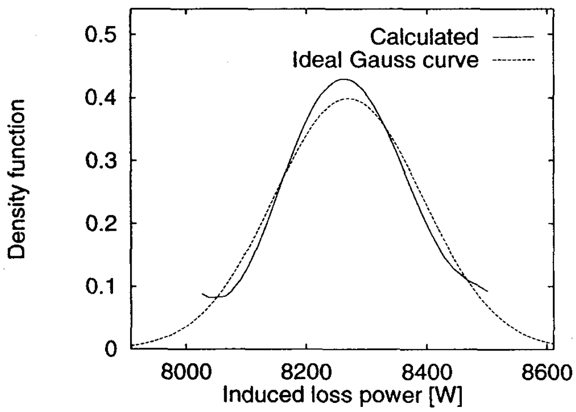


Fig. 8: Probability distribution density of 100 trials for f=20%

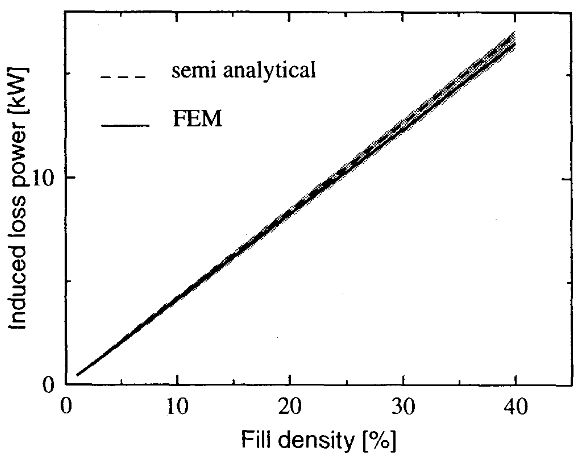


Fig. 9: Comparison of FE- and semi-analytical calculation

is used (fig. 6). The material distribution which is in fact non-axisymmetric can be considered approximately by the assumption that the material particles are not shorted.

The furnace was driven by a total current of 10000 A at 250 Hz. The ratio of particle perimeter to skin depth was 0.44 . For this model 1200 calculations have been done for both FE- and semi-analytical formulation.

The probability distribution for a fill density of 20% can be seen in fig. 7, where the FE calculated values, the spline interpolated ones and the ideal Gauss distribution for 100 trials are shown. By differentiation one gets the density function nearly as an ideal Gauss distribution (fig. 8).

The FE- and semi-analytical result agree very well for low fill densities but for higher values the shielding effects cannot be considered by the semi-analytical formulation.

IV. Conclusion

This paper shows a method for the calculation of eddy current loss of statistically distributed material. This distribution was randomly generated according to global measurable parameters. A large number of trials for one system state was produced, solved and evaluated by means of the probability theory. The resulting loss is not a sharp value but a mean value with a confidence interval.

The three loss calculation methods show good conformity. The $\vec{A}V$ formulation is generally faster than the integrodifferential approach. For low fill densities the fast semi-analytical approach is a good alternative for the loss calculation whereas for higher fill densities shielding effects lead to considerable differences from the FE calculations.

References

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