Axisymmetric Magneto-Mechanical Problem including Ferromagnetic Materials

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Abstract– The computation of the displacements of ferromagnetics as a consequence of magnetic fields is a theme of present research. In this paper a method of calculation is presented and the results are compared with measurements. The effect of a nonlinear material characteristic is examined. Its influence on the force distribution will be estimated.

I. INTRODUCTION

Within the scope of increasing sensitivity to noise emissions one pays more and more attention to magnetically stimulated vibrations and noises.

Magnetic fields produce surface forces on ferromagnetic material, such as laminations. If these forces happen to be variable in time, they cause dynamic deformation of the machine's surface and thereby an emission of noise. At present these facts are not completely quantifiable in the design of electric devices.[8]

The main problem lies in a correct model for ferromagnetic materials and in the calculation of the local force distribution, because an experimental verification of the force is not possible at the present time.

In this paper the displacement resulting from local forces will be calculated and compared to measurements. The computing process of the dynamic deformation is arranged in three steps. First, one calculates the magnetic field from the geometry, the material data and the exciting currents. Hence follows the determination of the local force distribution and from this the deformations. These deformations will be compared with gaugings of a test bench. In order to reach a good comparability with the measurements, an axisymmetrical layout of the test apparatus is chosen, which can be computed as a two-dimensional plane section.

II. CALCULATIONS

A. The Magnetic Calculations

The computations are based on the finite element method. Starting with Maxwell's equations

$$\nabla \times \vec{H} = \vec{J} \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

and the help of the vector potential $\vec{A}$ the stationary magnetic field problem is given by [1]

$$\nabla \times (\nu \nabla \times \vec{A}) = \vec{J} . \tag{3}$$

Here ν is the reluctivity and $\vec{J}$ the current density. Magnetization resulting from an alteration of the material's mass density during a distortion will not be considered.

B. The Force Calculations

The force calculation represents the coupling between the magnetic field and the displacements. Assuming the isotropy and homogeneity of the ferromagnetic material, the flux density can be represented as follows

$$\vec{B} = \vec{B}(\alpha_1, \alpha_2, \ldots, \alpha_n, \vec{H}) , \tag{4}$$

where the α_i are all parameters that influence the flux density, e.g. the permeability and mass density. The magnetic force density can now be calculated by the principle of virtual displacement. The differential energy stored in the magnetic field is equal to the mechanical work of a force $\vec{f}$ that extends the field area by a virtual displacement $\delta\vec{\xi}$

$$\int_V \delta w = - \int_V \vec{f} \cdot \delta\vec{\xi} . \tag{5}$$

For δw we get [2]

$$\delta w = \sum_{i=1}^{n} \frac{\partial w}{\partial \alpha_i} \delta\alpha_i + \frac{\partial w}{\partial \vec{B}} \delta\vec{B} . \tag{6}$$

After some steps of transformation according to [2] we find that the sum in (6) represents the material properties, while the second term stands for the energy density caused by the Lorentz force.

Finally the force density, caused by material properties only, can be written as

$$\vec{f} = \sum_{i=1}^{n} \frac{\partial w}{\partial \alpha_i} \nabla \alpha_i . \tag{7}$$

The transition to the Maxwell tensor $\bar{T}$ results in the following relation to the co-energy w' [2]

$$\bar{T} = \vec{H}\vec{B}^T - \mathbf{I}w' . \tag{8}$$

Here $\mathbf{I}$ is the unit matrix. Reducing the Maxwell tensor to a differential surface (Fig.1) one obtains the surface force density $\vec{\sigma}$

$$\vec{\sigma} = \lim_{d,l \to 0} \frac{1}{l} \oint_C \bar{T} \cdot \vec{n} \, da . \tag{9}$$

This leads to an expression for the surface force density

$$\vec{\sigma} = \Big(B_n (H_{1n} - H_{2n}) - (w_1' - w_2') \Big) \vec{n}_{12} . \tag{10}$$

It is obvious that the force density $\vec{\sigma}$ is always perpendicular to the surface.

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If tangential components were to appear, they would be caused by a violation of the boundary condition of the magnetic field.
Till now only the force density on surfaces has been considered. The internal forces are proportional [3] to grad μ. Nonlinear material characteristics lead also to a force distribution within the material.

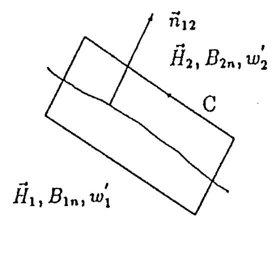

Figure 1: The marginal surface

To examine the magnitude of these internal forces the integral force is calculated. This integral force will be compared to the one calculated by the method based on virtual displacement [4]. The example's geometry is a two-dimensional c-core. The force distribution can be seen in Fig.2.

The surface forces at the air gap are obvious. The points within the material represent volume forces. Here the element edges have been seen as marginal surfaces to which (10) is applied.

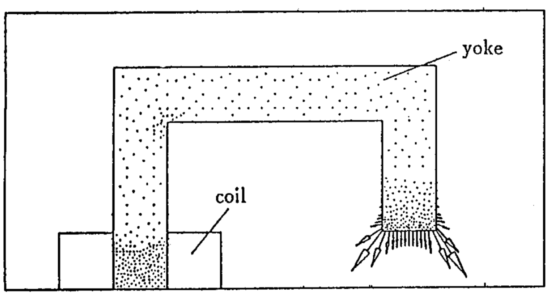

Figure 2: surface force densities

	calculation method	$F_y[N/m]$	$\Delta F_y[\%]$ with (1)	$\Delta F_y[\%]$ with (2)
(1)	virtual displacement	-1157.1	-	
(2)	linear characteristic	-1288.1	11.3	-
(3)	nonlinear characteristic without volume forces	-1300.8	12.4	0.98
(4)	nonlinear characteristic with volume forces	-1308.5	13.1	1.58

Table 1 : comparison of different force calculation methods

In Table 1 above, the global force is computed for an electric loading, that saturates wide areas of the core. The data in the table show that the solution for the core based on the virtual displacement calculation is about 12 % below the integral force density solution. The influence of the nonlinearity and especially the volume forces is about 1 % and therefore negligible.

C. The Displacement Calculation

The mechanical problem can be described by Hooke's material law

$$\underline{\sigma} = \underline{H} \cdot \underline{\epsilon} \tag{11}$$

and some equilibrium and boundary conditions. Here $\underline{\sigma}$ represents the mechanical tension, $\underline{H}$ Hooke's matrix and $\underline{\epsilon}$ the strain. Furthermore the connection between strain and displacement $\underline{f}$, which are coupled by the differential matrix $\underline{L}$, is of special interest

$$\underline{\epsilon} = \underline{L} \cdot \underline{f} \, . \tag{12}$$

The finite element equations for the displacement problem are derived according to [5] from a variational approach for the energy, which is called the elastic potential energy in this context. Neglecting initial tensions and strains and supposing that only surface forces Φ act on the magnetic material, one can write the whole elastic potential energy Π as

$$\Pi = \frac{1}{2} \int_V \underline{\epsilon}^T \underline{H}\, \underline{\epsilon}\, dV - \int_S \underline{f}^T \underline{\Phi}\, da \, . \tag{13}$$

In transition to finite elements the displacement will be described by the interpolation matrix $\underline{N}$ and the nodal displacement $\underline{d}$

$$\underline{f} = \underline{N} \cdot \underline{d} \, . \tag{14}$$

Using (13) with (12) and (14) and minimising the potential, one gets the following equation for the calculation of the nodal displacements

$$\underline{k} \cdot \underline{d} = \underline{r} \, . \tag{15}$$

Here $\underline{k}$ represents the so called element stiffness matrix, which comprises information about material and geometric

$$\underline{k} = \int_V (\underline{LN})^T \cdot \underline{H} \cdot (\underline{LN})\, dV \, . \tag{16}$$

and $\underline{r}$ stands for all loads on the element

$$\underline{r} = \int_S \underline{N}^T \cdot \underline{\Phi}\, da \, . \tag{17}$$

Considering Hamilton's principle [9], the whole stationary linear mechanical problem can be solved by the equation

$$(\underline{K} - \omega^2 \underline{M})\, \hat{\underline{D}} = \hat{\underline{R}} \, . \tag{18}$$

Here upper case letters are used to mark globalized matrices. ω is the mechanical angular frequency, which is twice the electrical frequency, and $\underline{M}$ is the mass matrix. With the help of that equation a very interesting aspect of the vibration analysis can be examined, that is the eigenvalues or resonant frequencies.

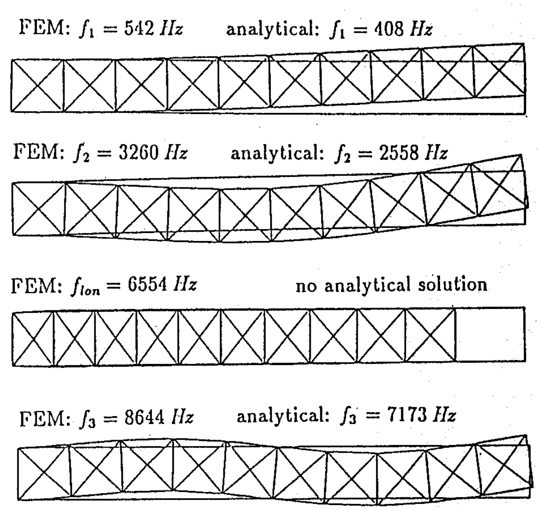

Figure 3: Eigenforms of a cantilever beam

As an example the eigenvalues and eigenvectors are determined for a cantilever beam and compared to results, based on a simplyfied analytical theory [6]. In Fig. 3 the three transverse and one longitudinal resonance are shown.

III. APPARATUS AND TEST BENCH

The apparatus has been chosen in such a way that the vibration generating processes are obvious. It consists of an air coil and a thin metal sheet fixed above it, which can easily be activated to vibrate. This sort of sheet is normally used for laminations in electrical machines.

The whole apparatus can be seen in Fig.4. The coil is supplied by an AC-converter, so that the frequency of the current can be varied over a wide bandwidth. The vibrations are measured by an accelerometer, the signals of which are Fourier transformed and finally integrated twice to get the displacements.

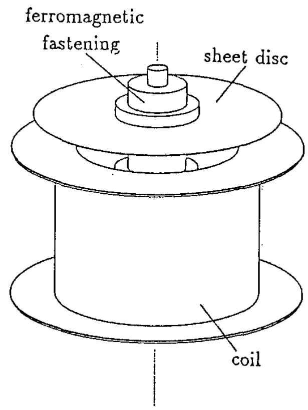

Figure 4: The apparatus

IV. RESULTS

Fig.5 presents the flux lines of the apparatus. Due to the sheet's high permeability nearly all of the flux is attracted by the sheet. The force density distribution on the sheet's surface can be taken from Fig.6. Here the resulting force density is the difference taken from the values of the upper and lower sides. The global force on the sheet is about 6 N.

The measurements were made at several sheets of the same material and geometry. These measurements showed a circumferential dependency, which can only be explained by the anisotropy of the sheets caused by rolling. The displacement oscillates with the circumferential angle und has its minimum in the rolling direction. For a mechanical frequency of 50 Hz one gets the following angle dependency of the displacement for two different sheets (Fig.7).

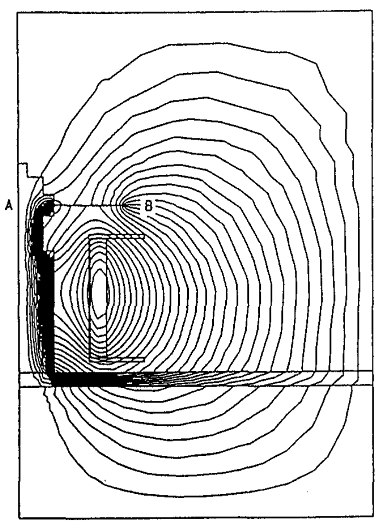

Figure 5: Equipotential lines

This three-dimensional effect overlaps with the axisymmetry of the apparatus, so that a comparison of the calculated values can only be drawn with an average measurement. Therefore the calculated resonance frequencies from the second on differ from the measured ones considerably.

Displacements of two sheets are shown in Fig.8. Different symbols mark several measurements and the dashed line their average. This is compared to the calculations, marked by the solid line. The computed displacement lies within the scattering band of the measurements and between the averages of the two sheets.

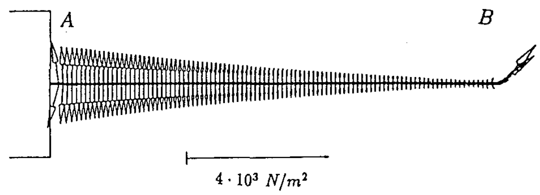

Figure 6: Force distribution

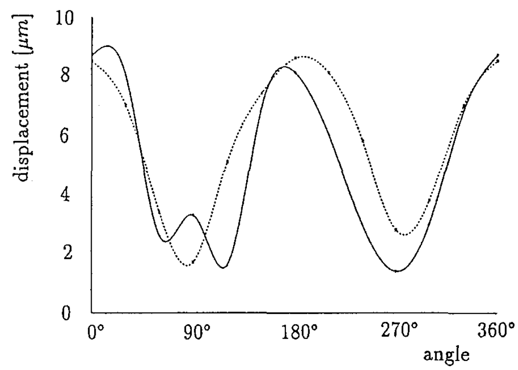

Figure 7: Anisotropy effects on the displacement

V. CONCLUSIONS

The presented procedure provides a numerical method of calculating vibrations of ferromagnetic material caused by time-varying magnetic fields. Starting from the geometry, material characteristics and currents displacements are calculated in three steps:

magnetic field ⇒ forces ⇒ displacements.

It has been shown that the magnetic forces within the material caused by nonlinear characteristic are very small compared with the surface forces.

The numerical results agree well with measurements made on a test bench. Further efforts to analyse three-dimensional effects and their influence on resonance frequencies should be made.

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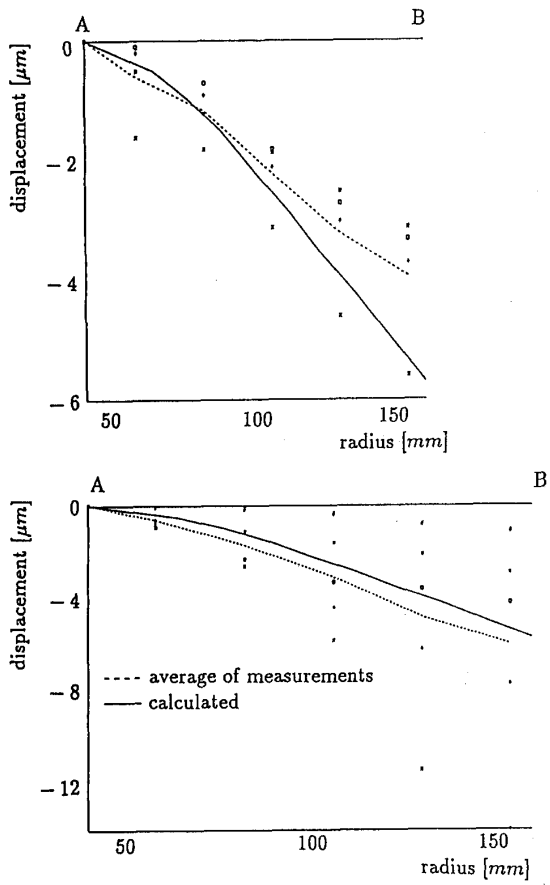

Figure 8: Comparison of calculated displacement and measurements

Biographical descriptions

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Elementary school in Essen from 1946 to 1950, secondary school in Mannheim from 1950 to 1959, final examination 1959.
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