NATURE OF THE EQUIVALENT MAGNETIZING CURRENT FOR THE FORCE CALCULATION

G. Henneberger, Ph. K. Sattler, D. Shen

Institute of Electric Machines
The Technical University of Aachen
Schinkelstrasse 4, 5100 Aachen, F. R. Germany

ABSTRACT

The method of the Equivalent Magnetizing Current is an alternative for the force calculation. The physical and the mathematical meaning is cleared. It is proved that the method is the same as the method of Maxwell stress for the total force, but different for the local force computation. Different computational procedures are analysed. Another procedure is proposed, which permits to compute either the total or local force with an improved accuracy.

INTRODUCTION

The force calculation becomes an important topic in the domain of the numerical computation of electromagnetic fields. Recently a number of publications have been reported about the accuracy and the reliability of different methods. Some new methods have been proposed. The methods for the force calculation can be classified into two groups. The first group contains the method of Maxwell stress and the method based on the virtual work principle. They are universal and can be used to compute the total force on either ferromagnetic or current-carrying objects. They have more a mathematical than physical meaning. The second group contains the method of the equivalent magnetic charge or current. They are in fact an extension of magnetic pole or Ampère-current model of ferromagnetic materials. They are very closely related to the macroscopic model of materials. The methods in two groups lead generally to good results under certain conditions.

Parallel to the development of the total force calculation one pays more and more efforts on the local force density computation. On the one hand not only the total force but also the local force is required in the design and optimisation of electric devices. A comprehensive understanding of the force distribution is helpful to improve the design. On the other hand the coupling between magnetic and mechanic, and sometimes also acustic problems becomes an actual development. It relates closely with vibration and noise problems in electric devices.

The computation of the local force distribution is a natural extension of the total force calculation. Almost all methods have their corresponding local force description. The method of Maxwell stress is once more a method one thinks at first. Maxwell stress is often understand as the magnetic pressure acting on the surface, then as the surface force density. In practice this method faces some numerical difficulties. If the integral path of Maxwell stress is chosen very closely to the iron-air surface, the error on the total force arises [2]. The sum of all surface forces may differ from the total force obtained on another path. The accuracy may be improved if the subdivision by the finite elements is refined. However it is a tedious task and one is not sure, what is a sufficient fine subdivision.

The approach of the equivalent magnetizing current provides another possible way to calculate the total force. As shown in [1] it is also applied to compute the local force. It relates to a material model, Ampère-current, and then provides a physical interpretation. In the following sections this method is presented and compared with the method of Maxwell stress. It will be cleared if this method can compute the local force. At last some numerical results are presented.

THE EQUIVALENT MAGNETIZING CURRENT (EMC)

The magnetic behaviour of a ferromagnetic material, hard or soft, can generally be described by

$$\vec{B} = \mu_0 \vec{H} + \vec{M} \tag{1}$$

where $\vec{B}$ is the flux density, $\vec{H}$ the field strength, μ_0 the magnetic permeability of vacuum, $\vec{M}$ the magnetisation. For a hard magnet in reversible region $\vec{M}$ can be considered as constant. From the view of macroscope $\vec{M}$ in soft material is induced by the external exciting field and $\vec{M}$ is a function of $\vec{H}$:

$$\vec{M} = (\mu_r - 1)\mu_0 \vec{H} \tag{2}$$

where μ_r is the relative permeability. As $\vec{M}$ and $\vec{H}$ are in the same direction this presentation is only valid for non hysteresis, isotropic materials. It is however a good approximation for the most used soft materials. For linear materials μ_r is a constant and for non linear materials μ_r is also a function of $\vec{H}$. In nonferromagnetic materials (e.g. in air) $\vec{M}$ vanishes.

The governing equation of a magnetic field is:

$$\vec{\nabla} \times \frac{1}{\mu_0}\vec{B} = \vec{J} + \frac{1}{\mu_0}\vec{\nabla} \times \vec{M}. \tag{3}$$

$\vec{J}$ is the conduction current density. The second term on the right side has the same effect as the conduction current $\vec{J}$ and for this reason it is called here the Equivalent Magnetizing Current (EMC) $\vec{J}_m$:

$$\vec{J}_m = \frac{1}{\mu_0}\vec{\nabla} \times \vec{M} \tag{4}$$

For linear materials $\vec{J}_m$ exists only on the boundaries.

The equation (3) leads to a very important result. The existance of magnetic materials can be replaced by the distribution of EMC. The real field is replaced by an equivalent field, in which there are no ferromagnetic materials, the conduction currents $\vec{J}$ and the equivalent magnetizing currents $\vec{J}_m$ are placed in vacuum ($\mu_r = 1$). Very useful Biot-Savart's law, which is only valid in an uniformed medium, can now be used. (With help of this law we can subtract the field of discretization currents, due to computational errors, from the computed field and improve considerably the accuracy [4].) It must be emphasized that such an equivalence is only valid for the field of the flux density $\vec{B}$. The flux density remains the same if magnetic materials are replaced by the EMC. To refind the field strength $\vec{H}$ however the original magnetic behaviour of materials (1) must be used, so the energy of the field remains no change.

Manuscript received July 7, 1991

0018-9464/92$03.00 © 1992 IEEE

FORCE CALCULATION BY TEH EMC

In the above section magnetic materials have been replaced by the equivalent magnetizing current (EMC) $\vec{J}_m$. The force on a magnetized body is now the force on $\vec{J}_m$. Generally the total force can be obtained by a form resembling the Lorentz force [5], that is

$$\vec{F} = \int_V (\vec{\nabla} \times \vec{M}/\mu_0) \times \vec{B}^0 dv + \int_S (-\vec{n} \times \vec{M}/\mu_0) \times \vec{B}^0 ds \qquad (5)$$

$\vec{B}^0$ is only one part of the field $\vec{B}$ and is the external field due to sources (conduction currents and magnetic materials) outside the body under consideration. Another part of the field $\vec{B}$ is $\vec{B}^1$, the self-field due to the magnetizing currents in the body. For low-frequency or stationary fields the self-force $\vec{J}_m \times \vec{B}^1$ must be zero. In equation (5) the term $\vec{\nabla} \times \vec{M}/\mu_0 = \vec{J}_m$ represents the EMC in the body and $\vec{J}_m \times \vec{B}^0$ the volume force density. For linear materials $\vec{M}$ is constant and the volume force vanishes. The second term is a surface integral. $-\vec{n} \times \vec{M}/\mu_0 = \vec{K}$ represents the EMC on the surface and $\vec{K} \times \vec{B}^0$ the surface force density.

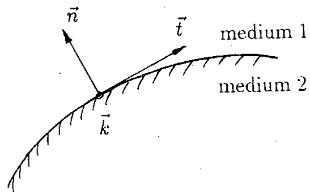


Figure 1. Boundary between two materials

In order to seperate the external field $\vec{B}^0$ from $\vec{B}$ consider a point on the surface of the magnetized body [1] (Figure 1). The body is in medium 2. Medium 1 is the air surrounding it. The external field is the same on both sides:

$$\vec{B}_1^0 = \vec{B}_2^0 \qquad (6)$$

The self-field of $\vec{K}$ is $\vec{B}_1^1$ and $\vec{B}_2^1$. Considering only the tangential component we have

$$\vec{B}_{t1} = \vec{B}_{t1}^0 - \vec{B}_{t1}^1 \qquad (7)$$

$$\vec{B}_{t2} = \vec{B}_{t2}^0 + \vec{B}_{t2}^1 \qquad (8)$$

The normal component of $\vec{B}^1$ is zero as $\vec{K}$ is a surface current. Noting that the self-field is identical on the both sides, because the material of the body has been replaced by EMC, and $\vec{K}$ is placed in vacuum, we have

$$\vec{B}^0 = \frac{1}{2}(\vec{B}_1 + \vec{B}_2) \qquad (9)$$

The equivalent magnetizing current on the surface is

$$\vec{K} = -\vec{n} \times \vec{M}_2/\mu_0 = M_{t2}/\mu_0 \vec{k} \qquad (10)$$

After (1) we have

$$\vec{M}_{t2} = \vec{B}_{t2} - \mu_0 \vec{H}_{t2} = B_{t2} - B_{t1} + \mu_0(\vec{H}_{t1} - \vec{H}_{t2}) \qquad (11)$$

Because $H_{t1} = H_{t2}$ the EMC on the surface can be represented by

$$\vec{K} = (B_{t2} - B_{t1})/\mu_0 \vec{k} \qquad (12)$$

The surface force (splitted up into the tangential and normal component) at the considering point is

$$\begin{aligned} f_t &= -KB_n^0 &&= -\frac{1}{\mu_0}(B_{t2} - B_{t1})\frac{B_{n1} + B_{n2}}{2} \\ f_n &= KB_t^0 &&= \frac{1}{\mu_0}(B_{t2} - B_{t1})\frac{B_{t1} + B_{t2}}{2} \end{aligned} \qquad (13)$$

Noting $B_{n_1} = B_{n_2}$ the above expression can be transformed into

$$\begin{aligned} f_t &= f_{t1} + f_{t2} &&= \frac{1}{\mu_0}B_{t1}B_{n1} - \frac{1}{\mu_0}B_{t2}B_{n2} \\ f_n &= f_{n1} + f_{n2} &&= \frac{1}{2\mu_0}(B_{n1}^2 - B_{t1}^2) - \frac{1}{2\mu_0}(B_{n2}^2 - B_{t2}^2) \end{aligned} \qquad (14)$$

Furthermore each component consists of two parts and the first part concerns only the field in medium 1. This part is exactly the same as Maxwell stress. The second part is Maxwell stress in medium 2, ferromagnetic material, but the magnetic behaviour (1) is not taken into account. It is Maxwell stress in air, not in materials. That leads to a very different result as that deriving from the direct application of Maxwell stress on both sides of the surface or from the energy method [3]. The force density for linear, homogeneous and isotropic materials after [3] is

$$\begin{aligned} f_t &= f_{t1} + f_{t2} &&= H_{t1}B_{n1} - H_{t2}B_{n2} \\ f_n &= f_{n1} + f_{n2} &&= \frac{1}{2}(H_{n1}B_{n1} - H_{t1}B_{t1}) \\ & && -\frac{1}{2}(H_{n2}B_{n2} - H_{t2}B_{t2}) \end{aligned} \qquad (15)$$

In most cases $\mu_r \gg 1$, so the second part of the force in (14) is μ_r times greater than that in (15). As this force part concerns only the field in medium 2, it represents the internal force, that is not interested for most applications. But this difference has no influence on the total force because the integral of the second force part on a closed, interior surface is zero. That will be proved in the next section.

Maxwell stress on the interior surface, the second force part in (15), is so small that it has practically no influence on the surface force. Another difference between (14) and (15) is that the tangential component of the surface force vanishes in Maxwell stress method. This component in (14) is however not zero. The existance of the tangential force means the distribution of magnetic particles (domain, Ampère-current, charge) is not stationary. From this point of view the method of Maxwell stress agrees better with the physical explanation.

After the above analysis the following conclusions can be obtained:

1. The method of EMC leads in theory to the same total force as the method of Maxwell stress. It is an alternative for the force calculation.
2. The method of EMC can however not be used to compute the local force. The second force part in (14) has another meaning in the numerical computation. That will be explained in the next section.

COMPUTATIONAL PROCEDURE

There exist three procedures to compute the force by means of the method of the EMC.

1st Procedure:[1]

1. compute after (12) the EMC on all element edges in the interesting region or object,

2. compute after (14) the force acting on these element edges,

3. add the forces in step 2.

As shown in (14) the force acting on each element edge has two parts. Each part concerns only the field in one of two elements, which are connected together by this element edge. So the two force parts can be computed seperately in two elements. Thus the computational procedure can also be described as: integrate Maxwell stress at the interior side of edges for all elements and at the exterior surface of the considering region. Inside of an element the integral of Maxwell stress at a closed contour is equal to zero, because there are no sources and the material is linear. The body force is then only the integral on the exterior surface. So the method of EMC is identical with the method of Maxwell stress.

In the linear case it is also evident that the integral of Maxwell stress on the interior surface of a magnetized body is equal to zero. The body force by integrating (14) or (15) is also identical with that obtained by the method of the magnetic pressure, the integral of Maxwell stress only on the exterior surface.

For linear materials the EMC vanishes on the element edges situated inside the body. The EMC exists only on the surface. So we have a second computational procedure.

2nd Procedure:[6]

- integrate after (14) the *force density* on the body surface.

Due to computational errors the integral of the second force part in (14) on the interior surface is not more equal to zero. But we have:

$$\int_{\Gamma} \vec{f}_2 ds + \int_{\Omega} \vec{f}_2 dv = 0 \qquad (16)$$

Γ is the interior surface and Ω are all element edges. (16) tells us that the integral of f_2 (the second force part in (14)) on the interior surface is equal to the negative value of the force on all element edges. The force on an element edge inside the body is understand as the volume force. The two elements connected by an element edge are in the same medium, the magnetized body. Look back to equation (5) the EMC on an element edge is

$$J_m = (M_{t2} - M_{t1})/\mu_0 = (\mu_r - 1)(H_{t2} - H_{t1}) \qquad (17)$$

J_m must be vanished in linear materials. In the formulation of the vector potential $H_{t1} = H_{t2}$ is however not verified. Due to this computational error the jump in $\vec{H}_t$ on element edges produces a nonvanishing current sheet, it is

$$J'_m = H_{t2} - H_{t1} \qquad (18)$$

The force on J'_m must be subtracted from the body force. But the second procedure subtracts the force on J_m, it is $(\mu_r - 1)$ times greater! In other words this procedure corrects $(\mu_r - 1)$ times over.

The force on J'_m can be computed as in (13)

$$\begin{aligned} f'_t &= -J'_m B^0_n \\ f'_n &= J'_m B^0_t \end{aligned} \qquad (19)$$

On the surface of the body the current sheet J'_m exists also. The force acting on it is not accounted in this procedure. So we propose the next computational procedure.

3rd Procedure:

1. compute Maxwell stress on the exterior surface,

2. compute after (19) the force acting on J'_m for all element edges in the considering body (also on the surface),

3. subtract the force obtained in step 2 from the forces in 1.

To compute the surface force only the element edges on the surface are considered. The error current J'_m on the surface is greater than that in interior.

NUMERICAL PRESENTATION

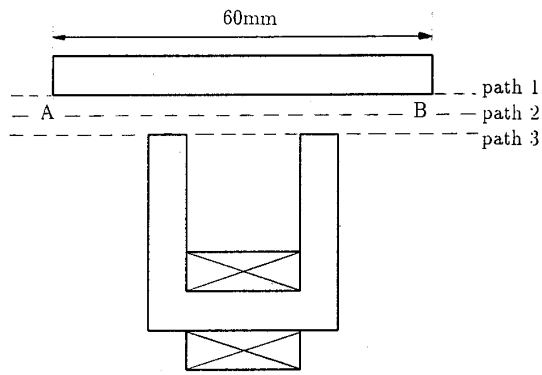


Figure 2. An air-gap electromagnet

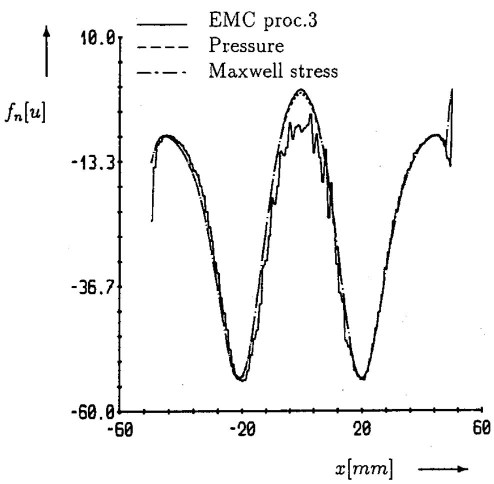
Consider the air-gap electromagnet shown in Figure 2. It consists of a U-shaped soft ferromagnetic material, which provides a flux path for the magnetic flux generated by the N turns of current I, and a keeper. We assume that the keeper and the U-shaped circuit have the same high permeability ($\mu_r = 1000$). An air gap δ exists when the circuit is not activated. The force density distribution on the low-side of the keeper, facing the electromagnet, will be computed. For the purpose of computation the force density is normed to

$$\frac{B_g^2}{2\mu_0} = \frac{\mu_0}{2}\left(\frac{NI}{2\delta}\right)^2 \qquad (20)$$

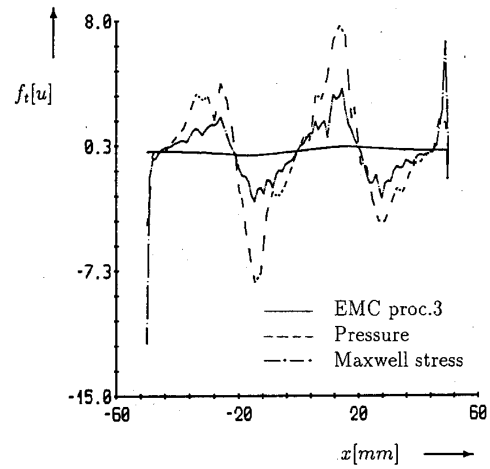
The force distribution for different methods is given in Figure 3. The method of the magnetic pressure (presented by *pressure* in Figure 3), Maxwell stress on the exterior surface, gives a force distribution which agrees very well with the physical image. Under a such force distribution the keeper has a tendency to bend. The method of Maxwell stress (presented by *Maxwell stress*) gives also a similar distribution. The second force part in (15) is very small and has practically no influence. The force distribution obtained by the EMC method, described in the third procedure (presented by *EMC proc.3*), is more accuracy because the force on the current sheet due to erors is subtracted. That will be proved in the total force calculation. The EMC method described in the second procedure gives a such force distribution which does not agree with the physical explanation. The value of the local force is also too high. The cause for that has been expained in the above section.

Table 1. The attractive force of the electromagnet

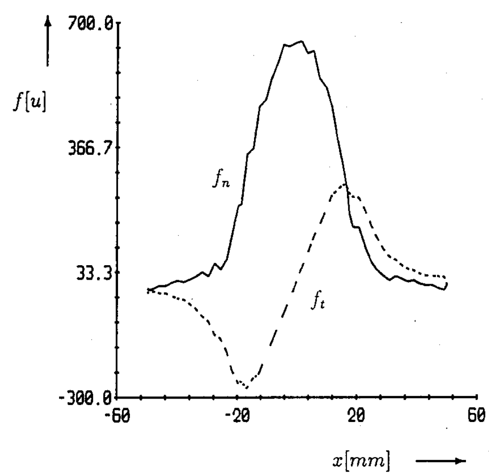
	coarse mesh	fine mesh
Pressure	-2.290	-2.373
Maxwell stress	-2.289	-2.372
Maxwell path 1	-2.381	-2.621
Maxwell path 2	-2.270	-2.339
Maxwell path 3	-2.361	-2.327
EMC proc.2	-1.822	-1.901
EMC proc.3	-2.292	-2.319



(a). local force distribution (normal component)



(b). local force distribution (tangential component)



(c). method of the EMC (proc.2)

Figure 3. The force distribution on the surface AB

The total force is given in Table 1 for two different meshes, coarse and fine. The force obtained by Maxwell stress on the path 2 (Figure 3), situated in the middle of the air gap, is taken as the reference. The method of the EMC (proc.3) has a very good result either in the coarse or fine mesh. Procedure 2 of the EMC method gives a too low force because the force is over corrected. The force obtained by the other methods differs from the coarse to fine mesh and has sometimes a different tendency (e.g. path 2). It is relatively sensible from the subdivision.

CONCLUSION

The method of the Equivalent Magnetizing Current (EMC) can be used to calculate the total force. It is in nature the same method as Maxwell stress, but it can not compute the local force. The integral of Maxwell stress on the interior surface represents the volume force. The method of the EMC subtracts the volume force due to computational errors from the force obtained by the method of Maxwell stress. The volume force is not correctly computed. Another computational procedure is proposed, which can compute either the total force or the local force. The accuracy of the force computation has then been improved.

REFERENCE

[1]. T.Kabashima et al. *Force calculation using magnetizing current* IEEE Trans. MAG-24, No.1, pp.451-454, Jan. 1988.

[2]. Mizia et al. *Finite element force calculation: comparison of methods for electric machines* IEEE Trans. MAG-24, No.1, pp.447-450, Jan. 1988.

[3]. G.Reyne et al. *A survey of the main aspect of magnetic forces and mechanical behaviour of ferromagnetic materials under magnetisation* IEEE Trans. MAG-23, No.5, pp.3767-3767, Sept. 1987.

[4]. G. Henneberger et al. *Achievement of analytical accuracy in the magnetostatic field computation by the finite element method*, CEFC'-90, Toronto, Canada, 1990.

[5]. W.F.Brown: *Magnetostatic principles in ferromagnetism*, North-Holland Publishing Co. Amsterdam, 1962.

[6]. T.Kabashima et al. *A Study on local force calculations*, Intermag'-89, 1989.