Assessment of Uncertainties and Tolerances in Electrical Machines

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A variety of manufacturing tolerances exists in electrical machines that influence their behavior. Accurate evaluation of the tolerances and uncertainties is important to assess the manufacturing quality of the machine. Electrical machines possess hundreds of tolerances that affect the machine differently, making evaluation a challenge. In order to assess the influence of manufacturing tolerances, high demands are placed on simulation accuracy, while a large number of simulations are required for the stochastic analysis of tolerances. The underlying simulation methodology can be extended from static FE simulations to complex transient drive simulations, including the use of model order reduction techniques. Therefore, a three-step sensitivity analysis is proposed to identify significant manufacturing tolerances on the machine output values. The analysis comprises a stepwise reduction of the simulated tolerances depending on their importance in order to quantify their influence and at the same time increase simulation accuracy. With this comprehensive analysis the most important tolerances and their exact influence on the machine are thus known. Results can be utilized to improve manufacturing quality and to reduce production waste. The resulting simulation data can also be applied to train a machine learning models to improve the detection of manufacturing defects.

***Index Terms*—Electrical machine, Sensitivity analysis, Tolerances, Parameter reduction, FE simulation, Drive simulation, End-of-line test.**

I. INTRODUCTION

THE modeling of electrical machines in design processes is often performed with ideal simulations. However, during manufacturing various tolerances and uncertainties arise that can influence the behavior and thus the quality of the machine. Resulting tolerances and uncertainties can trigger parasitic effects until the machine is considered as production waste, but a reduction of all tolerances would highly increase the production effort [1], [2]. Hence, an accurate sensitivity analysis identifies tolerances with a high influence on the machine output values from which conclusions for the design process and manufacturing can be drawn.

In several studies different approaches for sensitivity analyses (SA) are presented which are usually realized as One-factor-At-a-Time (OAT) analysis, where the influences of individual tolerances on different output values of the machine are analyzed. In the studies, the dynamic and static eccentricity, the width and height of the stator teeth, the dimensions of the permanent magnet and the remanence flux density are evaluated tolerances. As output values the torque, stator current spectra and the vibration spectra are analyzed [3]–[8].

By contrast, a thorough sensitivity analysis can be performed with a regression analysis from which the trend between tolerances and output values can be evaluated. Another approach is the analysis of variance (ANOVA) which cope interactions between tolerances as well as high-order effects that cannot be represented with an OAT analysis [9]. A few studies employ an ANOVA with the tolerances in electrical machines where the sensitivity indices are calculated with a metamodel [10]–[13].

The mentioned studies focus only on a few tolerances and output values, as the required model accuracy for the underlying simulations strongly increase the complexity, so that the computational effort for an extensive analysis is too high. Therefore, the exact influence of the tolerances on the output is not apparent and the appropriate conclusions cannot be drawn.

Thus, to overcome the low model accuracy and still include a variety of electrical machine tolerances in the analysis, a three-step sensitivity analysis is presented. The aim of the proposed methodology is to identify all significant manufacturing tolerances on the machine output values while maintaining high accuracy and model complexity at an affordable level.

As a first step of the proposed methodology, tolerances and their limits have to be evaluated by means of expert knowledge and Failure Mode and Effects Analysis (FMEA). Furthermore, quality objectives have to be determined from the output values that represent the quality of the machines. With the tolerances and quality objects an OAT analysis is conducted. The results are used to depict tolerances that can influence the quality objectives. In a second step, the tolerances are examined by a SA, which is based on a FE-simulation with ideal sinusoidal currents. As a result, the tolerances with the main influences on the quality objectives are identified. In the third step a transient drive simulation is applied for the sensitivity analysis to include transient effects of the inverter, control and drive train.

In section II potential tolerances of the electrical machine are discussed. Essential for the SA are the machine simulation methodologies which are described in section III. The assessment through the proposed sensitivity analysis methodology follows subsequently in section IV. Application scenarios for the results of the SA are presented in section V and remarks are drawn in section VI.

II. UNCERTAINTIES AND TOLERANCES IN ELECTRICAL MACHINES

In electrical machines there are various tolerances for rotor, stator and the assembly which can generally be divided into geometrical and material tolerances. Geometrical rotor tolerances include the outer radius r_R, the width h_M and height w_M

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of magnet pockets as well as flux barriers. Stator tolerances consist of the height h_T and width w_T of the teeth, the yoke and in case of a stator segmentation tolerances additionally the tolerances of the housing radius r_H as well as stator segmentation angles α_T.

The assembly of stator, rotor, bearings and housing components can also induce additional tolerances that affect the quality of the machine. For example, the static eccentricity $e_\mathrm{Ecc,S}$ with angle $\alpha_\mathrm{Ecc,S}$ can be described as distribution resulting from a tolerance chain of bearing clearance, bearing shield offset and housing bore offset. In contrast the distribution of the dynamic eccentricity consists of tolerances from the rotor shaft offset, rotor inner bore offset and rotor outer surface offset. Therefore, the tolerance count can quickly increase to hundreds of individual tolerances [14].

Material tolerances include hard and soft magnetic materials with individual tolerances. For example, magnets possess a remanence flux density distribution B_R as well as a temperature dependency. State-of-the-art in modeling of soft magnetic materials with FE analysis is an anhysteretic magnetization curve which cannot include physical material tolerances. To cover all material tolerance distributions that can influence the output of the machine, extensive material models with physical parameters have to be applied which also include additional tolerances by varying material compositions, rolling directions as well as steel cutting and lamination processes [15]. Thus, tolerance propagation of soft magnetic materials must be based on detailed models with physical material parameters.

III. EVALUATION WITH MACHINE SIMULATION

The machine simulation can be performed with different simulation approaches for considering the manufacturing tolerances. The accuracy of the simulation cannot be arbitrarily increased because the complexity and therefore the computational effort also increases disproportionately.

Analytical models are not suitable because geometrical tolerances are not applicable for estimating the influence on the output values. But a variety of methodologies for machine simulation can be considered, which are illustrated in Fig. 1.

The selection of the machine simulation methodology is depending on the simulation count, model complexity, model accuracy and the required quality objectives. A static 2D FE simulation is less complex than a transient drive simulation so that a larger number of tolerances can be included in the SA, but with the transient drive simulation sophisticated parasitic effects can be considered. The transient drive simulation consists of the control, power electronics, electrical machine and mechanic model. This allows to examine tolerances with parasitic effects from all components when a detailed sensitivity analysis is required. In most cases a weak field coupling with lookup tables is sufficient, but in some cases a strong field coupling with a FE simulation is useful, eg. when considering eddy currents. If only material tolerances at various operating points are in focus of interest, a Model Order Reduction (MOR) method can be employed to strongly reduce model complexity and therefore also decrease computational effort [16].

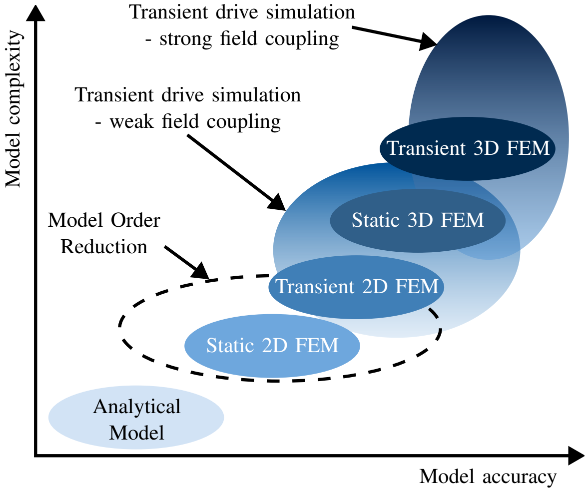

Fig. 1: Machine simulation methodologies.

The output values of the machine simulation cannot be directly utilized as sensitivity measures because single values are needed by SA but the output values are time- or space-dependent data series. Thus, the data series have to be condensed with different methods resulting in quality objectives, which are feasible as sensitivity measures. In general, the variance of radial forces σ^2_rad, which assess the radial force excitation on the stator without knowing the structural dynamics of the stator and housing, can be applied for all simulation methodologies [14]. The evaluation of the torque is dependent on the methodology: in an ideal FE simulation the torque is a direct output value but in a transient drive simulation the torque can be a controlled state variable, so that the mean torque $\overline{T}$ is equal for every calculated sample. In contrast, other output values such as the mean $\overline{I}$ or ripple current I_R can then be applied for the SA. In general, for an absolute analysis the quality objectives can be condensed with mean and ripple calculation for time- and space-dependent output values. In contrast, a frequency-based analysis can only be performed for a transient drive simulation, but then the Fast Fourier Transformation (FFT) can be applied to the output values, which allows an extensive analysis of the frequency orders and also the calculation of the stator surface acceleration a_eff.

As an example for SA, a PMSM with stator segmentation is used, which is also studied in [14]. The machine has three pole pairs with a rated power of 4.5 kW. The stator consists of fractional slot windings (number of holes $q = 0.5$) and the rotor poles are shaped as sinusoidal poles. The machine is utilized as pump drive and manufactured in a large-scale production. The operating point investigated is at rated speed $n = 4450\ \mathrm{rpm}$ and $T = 9.6\ \mathrm{Nm}$. All probability distributions are normal distributed, except for the bearing clearance, which is uniformly distributed.

IV. ASSESSMENT THROUGH SENSITIVITY ANALYSIS

A sensitivity analysis for stochastic studies consists of a Design of Experiments (DoE), a metamodel and the sensitivity indices [9]. The DoE describes with an algorithm the equally distributed tolerance parameter space whereas Sobol sequences

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converge faster with less samples than Monte Carlo methods. Also the transformation of equally distributed parameter spaces into unequally distributions, e.g. normal distributions, can be included in the parameter space. With the simulation data a metamodel is created from which the sensitivity indices are calculated. A Polynomial Chaos Expansion (PCE) metamodel represents the quality objectives and thus Sobol indices S_t can be calculated directly from the PCE coefficients, revealing the precise absolute influence of tolerances on the quality objectives which is based on an analysis of variance (ANOVA) as basic method [12]. The standardized regression coefficients β of the Linear Least Square (LLS) metamodel can reveal furthermore the trend between tolerances and quality objectives as regression analysis which is not possible with an ANOVA. Before analyzing the sensitivity indices, it is important to assess the goodness of prediction Q_2 of the metamodel. The goodness of prediction is based on the coefficient of determination R^2 [17]. The sensitivity measure is only reliable if the Q_2 coefficient shows a high prediction.

With the amount of individual tolerance parameters and the machine simulation methodology, a holistic sensitivity analysis cannot be performed because expressive conclusions cannot be drawn with hundreds of tolerances. Also the computational effort is barely manageable.

Therefore, a three-step SA to assess the uncertainties is proposed. The aim is to find the most influential tolerances of the electrical machine while reducing the tolerances in every analysis step. Concurrently, the model accuracy and complexity increases with each step.

The SA begins with the selection of tolerances that may have an influence on the quality objectives as pre-assessment. The second step consists of a SA with an ideal simulation to distinguish between less important and most influential tolerances. With the third step a precise SA containing a transient drive simulation is performed to estimate the precise influence of the most influential tolerances.

A. First Step – Pre-assessment

The first step (see Fig. 2) begins with the evaluation of potential tolerances and distributions from Failure Mode and Effects Analysis (FMEA) and expert knowledge. Afterwards the quality objectives, that are relevant to estimate the quality of the machine are selected. As first analysis, an OAT analysis is performed. As a result, the maximum and minimum influence of the tolerances on the quality objectives is calculated in comparison to a reference without tolerance deviations. With a defined absolute percentage limit the tolerance count is initially reduced. Furthermore, individual tolerances of a tolerance group have to be replaced with one tolerance parameter by means of stochastic tolerance patterns to assess the influence [14]. Otherwise there would be too many sensitivity indices for every tolerance group so that the overall influence for every tolerance group cannot be estimated.

For example, there are 116 individual tolerances from the FMEA for the PMSM, which probably influence the quality of the machine. With an OAT analysis by means of static 2D FE simulations 45 individual tolerances are identified with an influence of $> 1\%$ difference to the reference simulation. Considering stochastic tolerance patterns, the count is reduced to 10 tolerance parameters.

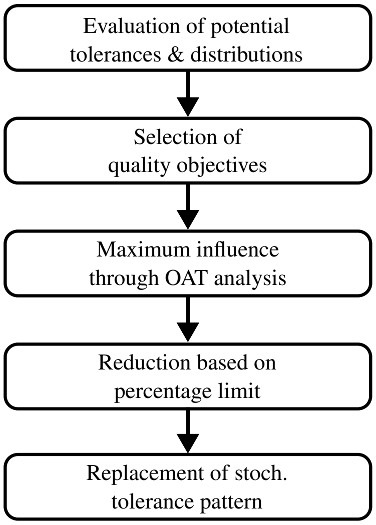

Fig. 2: First step: schematic of the pre-assessment.

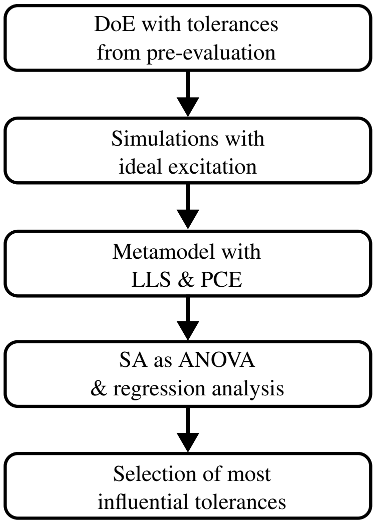

Fig. 3: Second step: schematic of the ideal SA.

B. Second Step – Ideal Sensitivity Analysis

The second step (see Fig. 3) begins with the creation of the tolerance parameter space through DoE with the size corresponding to the result of the previous step. Furthermore, the distributions of the tolerances are considered in the tolerance parameter space. The sample count depends on the computational effort and must be high enough so that the discretization of the parameter space does not affect the quality objectives. The simulation is performed with a lower model complexity to cover the computational effort for all samples, which consequently leads to a slightly lower accuracy of the quality objectives. Subsequently, the calculated quality objectives are represented by PCE and LLS metamodels from which the sensitivity indices are derived. The resulting SA reveals the exact influence of the tolerances with the Sobol indices of the

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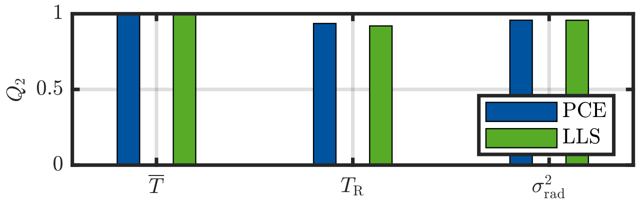
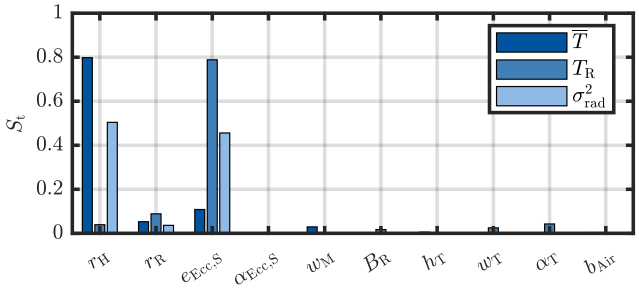
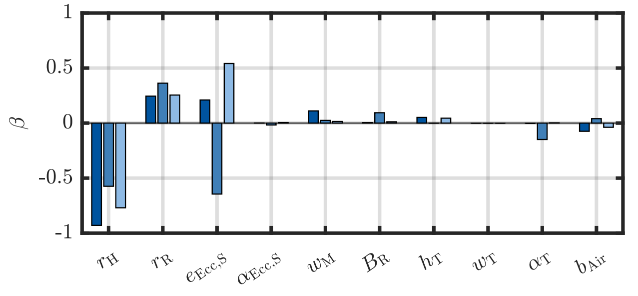


(a) Goodness of prediction



(b) ANOVA



(c) Regression analysis

Fig. 4: Sensitivity analysis of the second step with absolute quality objectives.

ANOVA and the trend with the regression analysis. The trend is important to estimate whether the tolerance correlates in a positive or negative direction with the quality objectives.

For the PMSM example, 600 samples and 10 tolerance parameters build the parameter space. As methodology a static 2D FE simulation with one rotor rotation is applied to simulate every sample. In Fig. 4 the results of the sensitivity analysis is depicted. The Q_2 coefficient reveals a precise representation of the quality objectives with the metamodel. As a result, tolerance groups have less influence than single tolerances. Here, the static eccentricity has the highest influence on the quality objectives. Furthermore, the regression analysis shows the trend that, for example, static eccentricity correlates negatively with torque ripple but positively with mean torque and variance of radial forces.

C. Third step – Transient Sensitivity Analysis

The tolerance count for the third step (see Fig. 5) depends on the computational effort of the utilized transient drive simulation. As in the second step, the samples of the tolerance parameter space have to be set in such a way that the discretization does not affect the quality objectives. Thus, the most influential tolerances have to be selected from the second step. The transient drive simulation is performed for all samples so that with the resulting time-dependent data absolute and frequency-based quality objectives are calculated, from which PCE and LLS Metamodels are created for absolute and frequency-based quality objectives. The subsequent SA is carried out for both types of quality objectives and conclusions are drawn for the design process, End-of-Line (EoL) test and

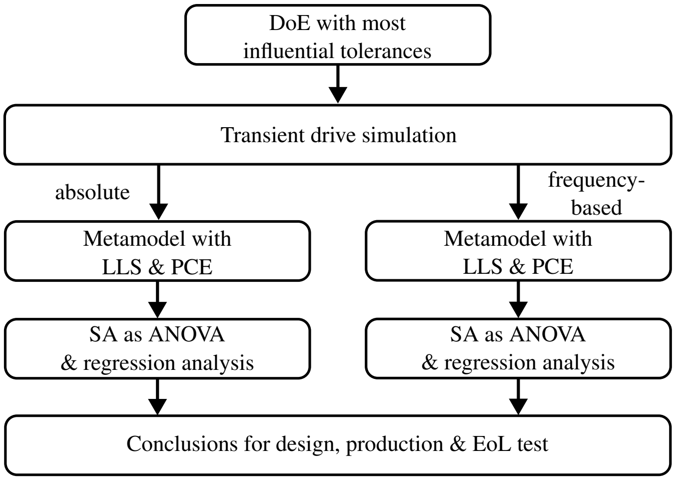
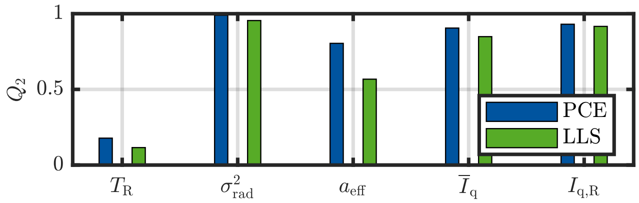
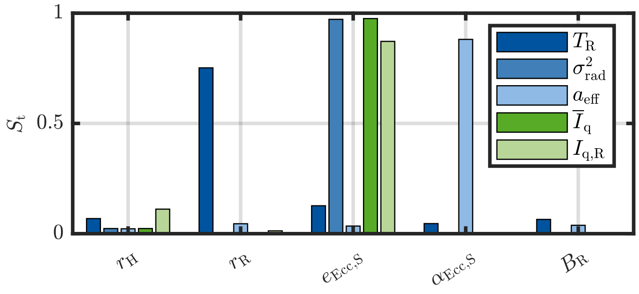


Fig. 5: Third step: schematic of the transient SA.



(a) Goodness of prediction



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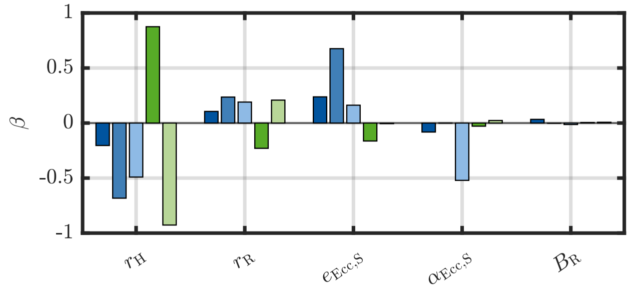

(c) Regression analysis

Fig. 6: Sensitivity analysis of the third step with absolute quality objectives.

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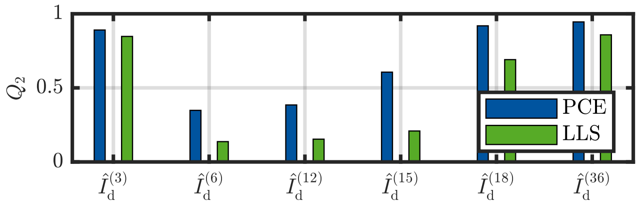
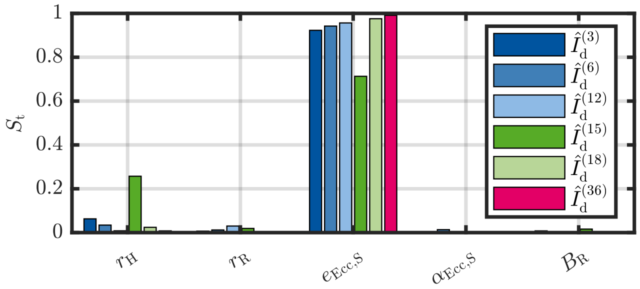


(a) Goodness of prediction



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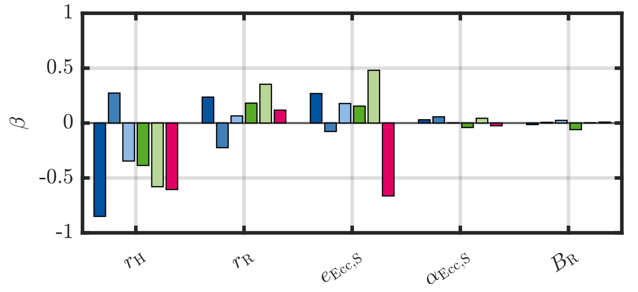
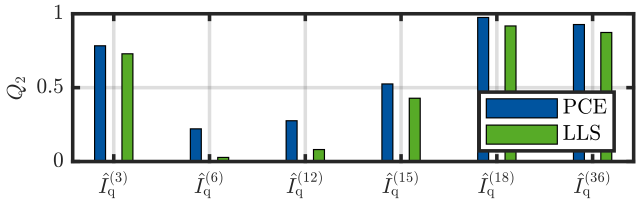
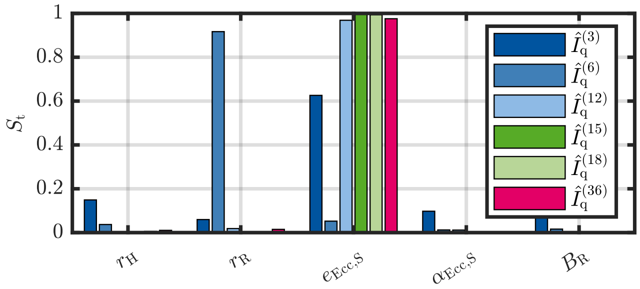


(c) Regression analysis

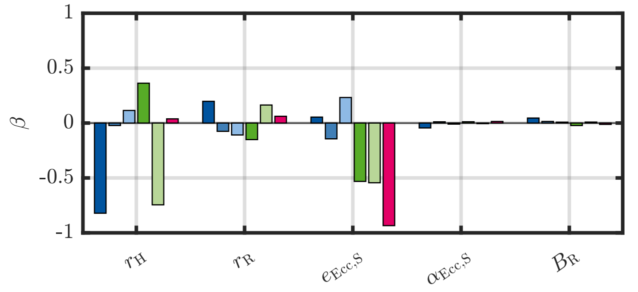
Fig. 7: Sensitivity analysis of the third step for the d-current frequency orders.



(a) Goodness of prediction



(b) ANOVA



(c) Regression analysis

Fig. 8: Sensitivity analysis of the third step for the q-current frequency orders.

Condition Monitoring (CM).

For the PMSM, the tolerance parameter space contains the five most influential tolerances from the second step with 200 samples. From the sensitivity indices based on the absolute quality objects overall conclusions can be drawn for the design process.

The absolute analysis in Fig. 6 also shows with the ANOVA that the static eccentricity has the highest influence on most quality objectives while the rotor radius only has a significant influence on the torque ripple. The goodness of prediction is adequately high for the quality objectives except for the torque ripple, so that this quality objective is only insufficiently represented by the metamodel. As a consequence, the influences of the tolerances on the torque ripple are not reliable. The trends between tolerances and quality objectives differ in direction, e.g. the housing radius has a negative influence on the variance of radial forces, but a positive influence on the mean q-current.

The frequency-based sensitivity analysis has to be split in separate analyses for each output value of the machine. The Q_2 coefficients in Fig. 7 and Fig. 8 for the d- and q-current frequency orders reveal, that orders 6, 12 and 15 are less reliable than the other frequency orders with respect to tolerance influences. The ANOVA of the d- and q-current frequency orders indicates that the static eccentricity has the most influence on the quality objectives, but some others are also slightly affected by the housing and rotor radius. The goodness of prediction for the torque frequency orders in Fig. 9 shows, that order 6 is only marginally represented by the metamodel. Likewise, the static eccentricity possess the highest influence on the torque frequency order 3.

The standardized regressions coefficients β for different frequency orders have varying trends and therefore reveal sophisticated influences of tolerances which can be utilized for further analyses, such as EoL tests and condition monitoring.

V. APPLICATION SCENARIOS

With the results of the SA conclusions can be drawn for the design process. In case of the discussed PMSM single tolerances have more influence on the quality objectives than tolerance groups. A simple improvement of the quality of the machine is the reduction of quality objectives by limiting the tolerances which is feasible for a small amount of high influential tolerances. An extended improvement is a multi-objective optimization of the quality objectives with boundary conditions which results in pareto-fronts to evaluate the optimal solution. In advance, the resource effort can be comprised into the objective function.

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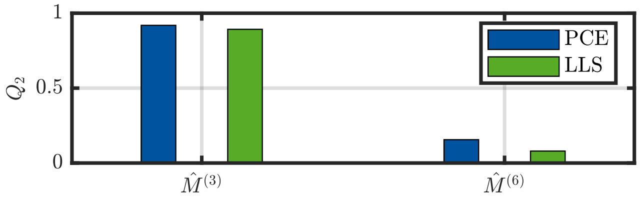
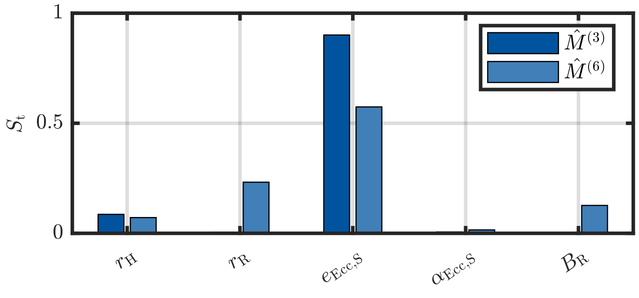


(a) Goodness of prediction



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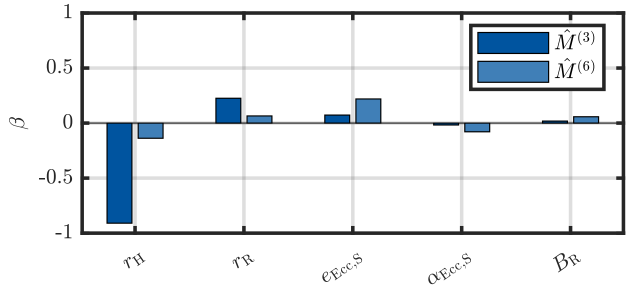

(c) Regression analysis

Fig. 9: Sensitivity analysis of the third step for the torque frequency orders.

Also the EoL test can be improved with a Machine Learning (ML) classification. Particularly, supervised ML methods can be applied on the simulation data for ML training. In case of the PMSM a predominantly reasonable classification with the simulation data of the third step is achieved. A fault recognition with an autoencoder functions without the need for fault data. Moreover, the autoencoder can also be trained with faultless measurement data from EoL tests.

VI. CONCLUSION

The consideration of tolerances and uncertainties in electrical machine is challenging because a high count of individual tolerances exists that influence the behavior and quality of the machine. The presented three-step sensitivity analysis is feasible for a thorough tolerance analysis to reduce the large computational effort while maintaining a high accuracy. The stepwise reduction of the tolerances is likewise essential for a precise analysis with extensive machine simulations. The results of the sensitivity analysis are strongly associated with the machine topology, design and manufacturing process.

In further research, sensitivity analyses for tolerances and uncertainties for different machine topologies have to be evaluated. Additional applications such as CM and EoL tests with ML support have a high potential in combination with the resulting simulation data and in focus of research.

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