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Analysis of the convergence of Newton method by finite element simulation with vector hysteresis stop model

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Abstract

Purpose – The goal of this research is to investigate the convergence behavior of the Newton iteration, when solving the nonlinear problem with consideration of hysteresis effects. Incorporating the vector hysteresis model in the magnetic vector potential formulation has encountered difficulties. One of the reasons is that the Newton method is very sensitive regarding the starting point and states distinct requirements for the nonlinear function in terms of monotony and smoothness. The other reason is that the differential reluctivity tensor of the material model is discontinuous due to the properties of the stop operators. In this work, line search methods to overcome these difficulties are discussed.

Design/methodology/approach – To stabilize the Newton iteration, line search methods are studied. The first method computes an error-oriented search direction. The second method is based on the Wolfe-Powell rule using the Armijo condition and curvature condition.

Findings – In this paper, the differentiation of the vector stop model, used to evaluate the Jacobian matrix, is studied. Different methods are applied for this nonlinear problem to ensure reliable and stable finite element simulations with consideration of vector hysteresis effects.

Originality/value – In this paper, two different line search Newton methods are applied to solve the magnetic field problems with consideration of vector hysteresis effects and ensure a stable convergence successfully. A comparison of these two methods in terms of robustness and efficiency is presented.

Keywords Magnetic hysteresis, Finite element analysis, Vector stop hysteresis model

Paper type Research paper

1. Introduction

Hysteresis models in a dual representation of the magnetic field are particularly well-suited models to analyze soft magnetic materials in the magnetic vector potential formulation, which is used in the finite element (FE) analysis. The vector stop model belongs to this family of hysteresis models and is accurate in resolving the anisotropy, the hysteresis losses and the hysteresis saturation properties of ferromagnetic materials (Matsuo *et al.*, 2004; Leite *et al.*, 2005). Incorporating a hysteretic material model, such as the vector stop model, into the FE analysis requires an iterative process

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to resolve the nonlinearity. Based on the Taylor series expansion, the magnetic vector potential is derived by solving the Jacobian matrix and the residual of the equation system. Starting from the initial guess, the equation system is iteratively solved toward the real solution by minimizing the residual. In each iteration step, the material model needs to provide the symmetric and positive definite differential reluctivity tensor for the Jacobian matrix and the magnetic field strength $\boldsymbol{H}(\boldsymbol{B})$ for the residual based on the magnetic flux density $\boldsymbol{B}$. Deriving the differential reluctivity tensor $\boldsymbol{v_d}$ tensor is difficult or even impossible, as the material model may not be continuously differentiable (Chama *et al.*, 2018). To overcome this problem, an equivalent representation of the differentiation is implemented and discussed in (Mitsuoka *et al.* 2013). As the Newton method is sensitive to the derivative, the insufficient $\boldsymbol{v_d}$ may lead to divergence (Dlala *et al.*, 2008). Furthermore, the initial guess point of the Newton iteration should be close to the solution point; otherwise, the approach may not be stable and fail to converge (Ahookhosh and Ghaderi, 2017).

The vector hysteresis stop model presented in this work is constructed with stop operators. Each of the stop operators has two magnetization states according to the input magnetic flux density and the threshold value of the corresponding operator. Therefore, the derivative of $\boldsymbol{H}(\boldsymbol{B})$ is discontinuous, as the operator jumps between the two states. The Newton method is not suited to solve this problem as previously presented, but Fujiwara *et al.* (1993) presented an optimal relaxation method to prevent the Newton steps from divergence. This optimal relaxation method based on residual monotonicity test is also used in the previous work (Guérin *et al.*, 2017) to deal with the Jiles Atherton's model. In this contribution, another relaxation method based on natural monotonicity test (Deuflhard, 2011) is used to stabilize the iterative solving process. The convergence can be achieved when the natural monotonicity test is fulfilled with the relaxation factor. Besides this relaxation method, another inexact line search method based on the Wolfe-Powell rule is implemented. The first condition of Wolfe-Powell rule is the Armijo condition, which is supposed to ensure a sufficient descent. The second condition named curvature condition, which ensures a sufficiently large update of the step size. This is very essential, as too small damped factor restricts the convergent progress.

2. Study of convergence behavior of the Newton method with vector hysteresis stop model

The n^{th} stop operator $\boldsymbol{S}_n$ of a vector hysteresis stop model based on the construction from the study of Leite *et al.* (2005) can be written as:

$$\boldsymbol{S}_n = \boldsymbol{B}_{\text{re}}{}^{t} = \begin{cases} \boldsymbol{\Omega}_n & \text{if } \ |\boldsymbol{R}_{\text{temp},n}{}^{-1}\boldsymbol{\Omega}_n| < 1 \\ \boldsymbol{R}_{\text{temp},n}\dfrac{\boldsymbol{\Omega}_n}{|\boldsymbol{\Omega}_n|} & \text{if } \ |\boldsymbol{R}_{\text{temp},n}{}^{-1}\boldsymbol{\Omega}_n| \geq 1 \end{cases} \tag{1}$$

where $\boldsymbol{\Omega}_n = \mathrm{d}\boldsymbol{B} + \boldsymbol{S}_n{}^{t-1}$, $\mathrm{d}\boldsymbol{B} = \boldsymbol{B}^t - \boldsymbol{B}^{t-1}$ and $\boldsymbol{R}_{\text{temp},n}$ is the threshold diagonal matrix. $\boldsymbol{R}_{\text{temp,n}}$ describes the threshold values in x and y directions of the n^{th} stop operator. More details about the construction of the threshold surface are presented in the previous work (Xiao *et al.*, 2022). The parameter $\boldsymbol{B}_{\text{re}}{}^{t}$ describes the reversible part of the magnetic flux density of time step t. The formal definition of a stop operator in equation (1) implies that the system can be divided into an elastic state and a plastic dissipated state. When the input is restricted in the range that $\boldsymbol{\Omega}_n < \boldsymbol{R}_{\text{temp,n}}$, the system possesses only an elastic response and is equal to the corresponding anhysteretic evolution $\mathrm{d}\boldsymbol{B} + \boldsymbol{S}_n{}^{t-1}$. When the input exceeds the threshold value $\boldsymbol{R}_{\text{temp,n}}$, the state of the system is described by the dissipated dry friction element. The stop operator is updated with value of the threshold and the direction of the history $\boldsymbol{\Omega}_n$.

To demonstrate the differentiation of the stop model, an alternating excitation from –1.6 T to +1.6 T in 30° is applied to the model. Based on the stop operators, the vector hysteresis stop model is further constructed with the anhysteretic surfaces $\boldsymbol{H}_{\text{anhys}}(\boldsymbol{B})$ and the weight w_n of the n^{th} operator $\boldsymbol{S}_n$:

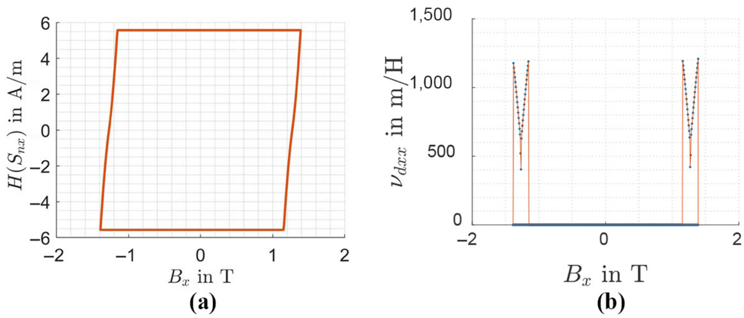
$$\boldsymbol{H}(\boldsymbol{B}) = \sum_{n=1}^{N} w_n \boldsymbol{H}_{\text{anhys}}(\boldsymbol{S}_n) \tag{2}$$

A graphical illustration of the interpolated values $\boldsymbol{H}_{\text{anhys}}(S_{nx})$ in x direction of n^{th} stop operator with constant threshold values is shown in Figure 1(a). As long as the excitation, which evaluated as $\boldsymbol{\Omega}_n$ from the time step t, is smaller than the threshold value $\boldsymbol{R}_{\text{temp},n}$, the $\boldsymbol{S}_n$ keeps updating with the value of $\boldsymbol{\Omega}_n$. This leads to the left and right curves in Figure 1(a). Until the input $\boldsymbol{\Omega}_n$ exceeds the threshold value, the $\boldsymbol{S}_n$ suddenly remains at the value of $\boldsymbol{R}_{\text{temp},n}$. This activation of the operator results in the upper and lower horizontal lines, as shown in Figure 1(a). The differential reluctivity tensor in equation (3) is calculated by automatic differentiation of the anhysteretic surfaces. A schematic representation of the anhysteretic surfaces and differential reluctivity tensor surfaces can be found in Xiao *et al.* (2021):

$$\mathbf{v}_{\mathbf{d}} = \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{B}} = \begin{bmatrix} \dfrac{\partial H_x}{\partial B_x} & \dfrac{\partial H_x}{\partial B_y} \\ \dfrac{\partial H_y}{\partial B_x} & \dfrac{\partial H_y}{\partial B_y} \end{bmatrix} \tag{3}$$

Once the operator is activated, the $\boldsymbol{v_d}$ results in a jump, which can be observed in Figure 1(b). The $\boldsymbol{H}(\boldsymbol{B})$ is constructed with parallel connections of the stop operators. The differential reluctivity tensor $\boldsymbol{v_{dxx}}$ of the stop model under one periodic alternating excitation in 30° is exhibited in Figure 2. In the subfigure of Figure 2(b), the jump from the stop operator can be observed in the $\boldsymbol{v_{dxx}}$ of the vector hysteresis stop model.

The discontinuity in $\boldsymbol{v_d}$ causes an insufficient Jacobian matrix for the Newton method, which could lead to the instability or even the divergence of the Newton iteration.



Source: Own simulation

Figure 1. (a) The x component of the from anhysteretic surfaces interpolated stop operator $\boldsymbol{H}_{\mathbf{anhys}}(\boldsymbol{S}_{\mathbf{nx}})$ and (b) the evaluated differential reluctivity tensor in x direction $\mathbf{v}_{dxx} = \frac{\partial \boldsymbol{H}_{\mathbf{anhys}}(\mathrm{S}_{\mathrm{nx}})}{\partial \boldsymbol{B}_x}$

3. Line search methods for field problems with vector hysteresis stop model

Considering the nonlinear magnetic static case, ΔA_k of k^{th} Newton iteration is solved with the Jacobian matrix $J(A_k)$ and the residual $R(A_k)$:

$$J(A_k)\Delta A_k = -R(A_k) \tag{4}$$

To save the Newton iteration from instability and extend the range of convergence, line search methods are introduced. The line search algorithm chooses a direction d_k to search and finds a relaxation factor α_k to move along with one searching step, which minimizes the objective function (Nocedal and Wright, 2006). In our case, the object function is the residual $R(A_k)$. The line search algorithm creates a series of α_k and terminates the searching when a certain condition is satisfied by the k^{th} searching step. The α_k is then accepted to update the solution by:

$$A_{k+1} = A_k + \alpha_k \Delta A_k, \qquad \alpha_k \in (0, 1] \tag{5}$$

3.1 Line search with nature monotonicity condition

The condition for the first line search method is chosen as natural monotonicity test, as this error-oriented framework aims at overcoming the ill-conditioned Jacobian matrix (Deuflhard, 2011). The natural monotonicity test expects the error ΔA to fall monotonically as formulated by:

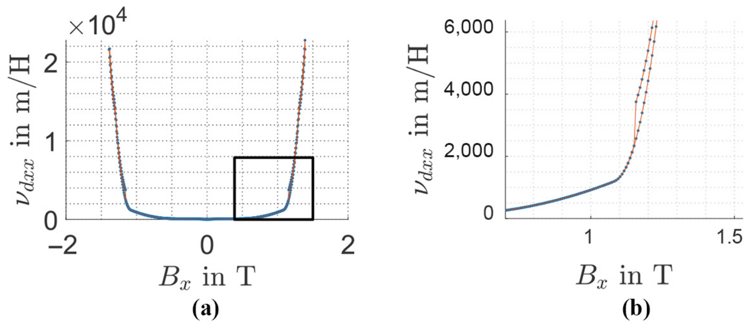
$$||\Delta A_{k+1}|| \;\leq\; \theta||\Delta A_k|| \tag{6}$$

where $\theta < 1$. Thus, (6) can be rewritten as (9) by using the simplified Newton corrections (7) and (8) from (Deuflhard, 2011):

$$||\Delta A_{k+1}|| \;=\; ||J(A_k)^{-1}R(A_{k+1})|| \tag{7}$$

$$||\Delta A_k|| = ||J(A_k)^{-1}R(A_k)|| \tag{8}$$

$$||J(A_k)^{-1}R(A_{k+1})\;|| \leq \;\theta||J(A_k)^{-1}R(A_k)|| \tag{9}$$



Source: Own simulation

Figure 2. (a) Differential reluctivity tensor $\boldsymbol{\nu_{dxx}}$ of the stop model and (b) enlarged in the range of the $\boldsymbol{\nu_{dxx}}$ where the jump occurs

The θ is chosen as $1 - \frac{\alpha_k}{2}$ and the (9) can be formed as:

$$\|J(A_k)^{-1}R\left(A_k + \alpha_k^i \Delta A_k\right)\| \leq \left(1 - \frac{\alpha_k^i}{2}\right)\|J(A_k)^{-1}R(A_k)\| \tag{10}$$

For the i^{th} damped step, if this nature monotonicity test (10) is fulfilled, the damped process is stopped, and α_k^i is set into (5) to calculate A_{k+1}. For the next Newton iteration, the first line search step starts with the damped factor $\alpha_{k+1}^1 = \min\left(2\alpha_k^i,\ 1\right)$ to accelerate the convergence. If (10) fails, α is updated further with:

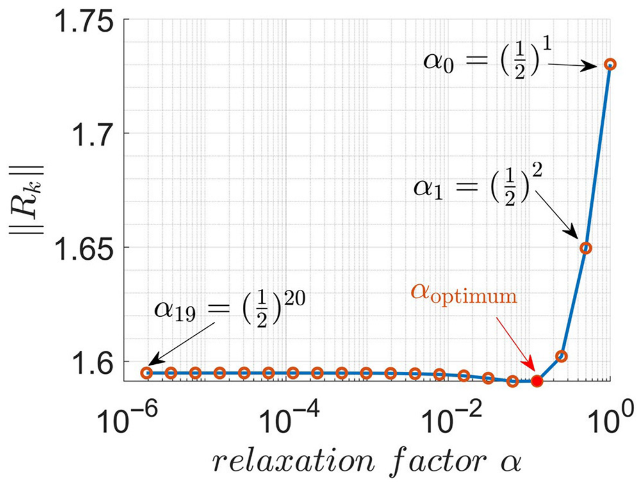
$$\alpha_k^{i+1} = \frac{\alpha_k^i}{2} \tag{11}$$

To illustrate the convergence process of the line search iteration by simulation with consideration of vector hysteresis effects, the residual of the equation system is calculated in each line search step. The vector stop model constructed with two stop operators is able to represent the anisotropic and saturation properties of the magnetic materials.

The development of the object function against the α is shown in Figure 3. To illustrate the process of line search iteration, the maximum number of damped steps is restricted to 20. With the monotonicity test (10), the optimal α can be found at the fourth step inside of the second Newton iteration. With this optimal damped factor α, the norm of the residual reaches the minimum value.

3.2 Line search with Wolfe-Powell condition

The step size of the line search method s_k depends on the direction d_k and the damping factor α_k, which is formed as:



Note: The found optimal relaxation factor by monotonicity condition is 0.125 at the fourth damped iteration
Source: Own calculation

Figure 3. The development of the residual $\|\boldsymbol{R_k}\|$ against the relaxation factor α by a second time step and second Newton iteration

$$s_k = \alpha_k d_\mathrm{k} \tag{12}$$

Therefore, to ensure a successful line search iteration, it is essential to choose a proper direction and a reasonable damping factor.

With the Armijo condition:

$$R(A_k + \alpha_k \Delta A_k) - R(A_k) \le c_1 \alpha_k J(A_k)^T \Delta A_k \tag{13}$$

of the Wolfe Powell rule, a damping factor $\alpha \in (0, 1]$ is accepted, when the step size s_k enables a sufficient descent.

The $c_1 \in (0, 1)$ is usually chosen to be small value as 10^{-4}. The right-hand side of equation (13) is a linear function and has a negative slope $c_1 J(A_k)^T \Delta A_k$ with respect to α_k (Nocedal and Wright, 2006). This condition determines a maximum step size and ensures a descent from last iteration step. However, only imposing the Armijo condition is not sufficient, as the damping factor could be chosen at too small values. The line search progress is therefore stuck in the range of starting point when α_k is close to the left-hand extreme 0. To ensure a satisfied progress, the curvature condition (14) is added:

$$J(A_k + \alpha_k \Delta A_k)^T \Delta A_k \ge c_2 J(A_k)^T \Delta A_k \tag{14}$$

where $c_2 \in (c_1, 1)$. Choosing the direction of Newton method as searching direction, the typically value of c_2 is chosen as 0.9. The left-hand side of equation (14) indicates that the slope of the next iteration step should be greater than c_2 times initial slope of this iteration step (Nocedal and Wright, 2006). The Armijo condition together with the curvature condition are referred to as Wolfe-Powell condition (Fletcher, 2000).

The strong Wolfe-Powell condition needs the damped factor αk to satisfy the condition (15) instead of (14):

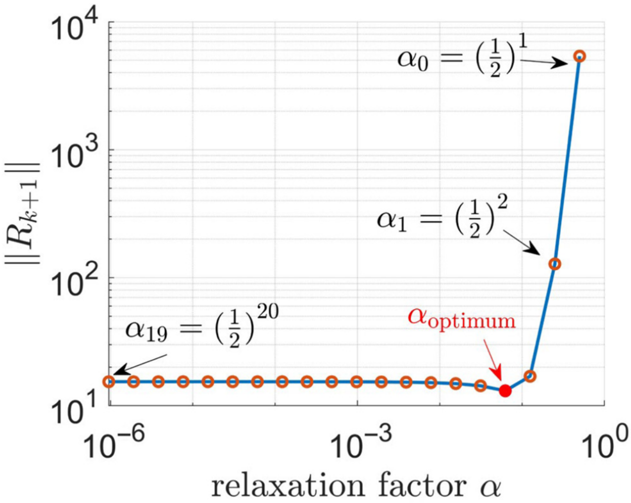
$$\| J(A_k + \alpha_k \Delta A_k)^T \Delta A_k \| \le \| c_2 J(A_k)^T \Delta A_k \| \tag{15}$$

which exclude the points far from the stationary point of $R(A_k + \alpha_k \Delta A_k)$ (Nocedal and Wright, 2006).

Applying the strong Wolfe-Powell condition to solve the field problem, the convergence progress against the damped factor is shown in Figure 4.

3.3 Numerical studies with line search methods

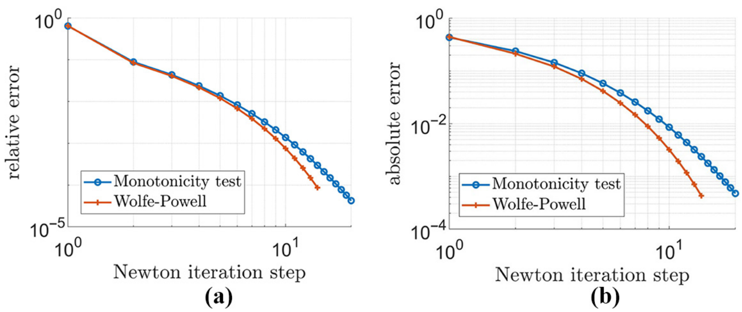
The line search damped Newton methods are further applied to solve the field problems. Numerical studies based on teamwork problem 32 (Bottauscio *et al.*, 2002) are carried out. The parameters of the vector stop model are identified with the given magnetic measurements. To demonstrate the behavior of the convergence in difficult situations, the test case 2 with harmonic excitation is studied. The relative and absolute tolerances of the Newton iteration are set at 0.5×10^{-3}. Calculating with the classic Newton method, the iteration process goes divergent. To save the process from divergence, both line search methods are applied to solve the field problem. The convergent progress with respect to the relative and absolute errors of the both line search methods is illustrated in Figure 5. This figure clearly demonstrates that the line search with Wolfe Powell rule has a faster convergent process in comparison with the line search iteration based on monotonicity test.



Note: The found optimal relaxation factor by Wolfe-Powell condition is 0.125 at the fourth damped iteration

Source: Own calculation

Figure 4. The development of the residual $\|\boldsymbol{R}_{k+1}\|$ against the relaxation factor α at the first time step and the first Newton iteration



Notes: (a) Relative error of the Newton iteration; (b) absolute error of the newton iteration

Source: Own calculation

Figure 5. Comparison of the convergence by the line search methods based on monotonicity test and Wolfe-Powell condition at the third time step by test case 2

In the study case 2, the simulated results within one period of time is shown in Figure 6. The simulated magnetic flux density in y direction on C6 with both line search methods is compared with measurements. In Figure 6(a), the results obtained from both methods are almost identical. In comparison with measurements, it is clearly seen that the computed field trajectories exhibit a better agreements in the hysteresis major loop than in the minor loops. This could be observed in the Figure 6(a) that the minimum and maximum values of flux density is well modeled with the proposed approach, whereas significant deviations in local extremes are shown. The reason for the inaccuracy in minor loops could be induced from the insufficient precision by modeling the minor loops under 0.5 T with only two stop operators of the vector stop model. To improve the accuracy by minor loops, the vector stop model should be constructed with more stop operators.

During the simulation, one of the problems is observed from the simulation with monotonicity test. This method is sensitive to the initial guess of the iteration, and the relative error could be reduced too fast with the damped factor, which restricts the convergence process. When zooming in on the time range from 0.09 s to 0.096 s, the insufficient convergence of the simulation with monotonicity test can be observed in Figure 6(b). The reason is that the damped factor is reduced to a very small value, which induces the very fast decreasing of the relative error of the Newton iterations. In the meanwhile, the absolute error of the Newton iteration still stays on a value far away from the absolute tolerance. The Newton iteration is thus restricted, and the convergence process gets stuck. The corresponding relative error and absolute error by the time of 0.0925 s calculated with the both line search methods are shown in Figure 7.

4. Conclusions and future work

Due to the properties of the stop operators, the jump discontinuity occurs in the differential reluctivity tensor when the state of the stop operator switched. This discontinuity could lead

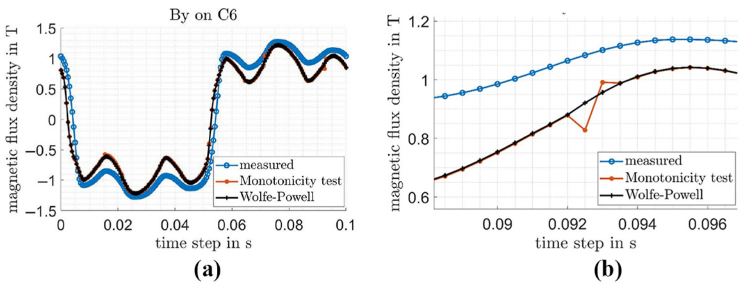


Figure 6. (a) Comparison of the measured magnetic flux density in the y direction on the pick-up point C6 within one period with the simulated magnetic flux density by using monotonicity and Wolfe-Powell line search Newton methods and (b) zoomed in the range where the insufficient convergence occurs by using the monotonicity test

Source: Own calculation

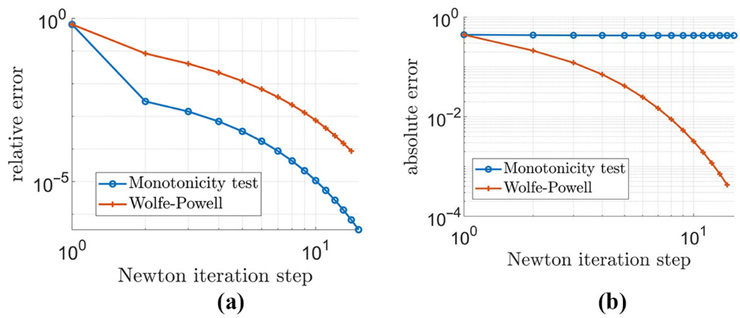


Figure 7. (a) Relative error of the Newton iteration and (b) absolute error of the Newton iteration

Source: Own calculation

to the divergence of the Newton iteration. In this paper, line search methods are proposed in combination with an improved differentiation of the vector stop model to ensure a good convergence. The insufficiency of the method with nature monotonicity condition is pointed out. A more efficient and robust simulation can be ensured by the line search method with Wolfe-Powell condition. The presented approach stabilizes the Newton method and presents a further step into accurate consideration of ferromagnetic hysteresis effects in the magnetic analysis of e.g. electrical machines. The line search methods with Wolfe-Powell condition could prevent the Newton iteration with insufficient Jacobian matrix and possible improper initial guess from divergence.

In computational magnetics, the trust region method is also a popular method to promote the convergence of nonlinear iteration. This method will be studied in future work. To achieve a higher accuracy by representing the minor loops, the vector stop model constructed with more stop operators can be applied to the FE simulation.

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