Nonlinear Parametric Simulation by Proper Generalized Decomposition on the Example of a Synchronous Machine

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***Abstract*— The acurate simulation of electrical machines involves a large number of degrees of freedom. Particularly, if additional parameters like remanence variations or different operating points need to be analyzed the computational effort fastly increases, known as the "Curse of Dimensionality". To cope with this effort the Proper Generalized Decomposition is employed incorporating additional excitation parameters into the reduced order model.**

Keywords—Model Order Reduction, Proper Generalized Decomposition, DEIM, Synchronous Machine

I. INTRODUCTION

The reduction of computational effort related to the numerical simulation of electromagnetic machines is a topic of increasing relevance. In the design stage many parameter combinations need to be analysed. In combination with the degrees of freedom (DOF) resulting from the spatial discretization of the Finite Element Method (FEM) the computational effort increases fastly. Model order reduction techniques cope with this problem, by employing approximations of the solution in a reduced subspace [1-6]. The Proper Generalized Decomposition (PGD) is particularly suited for the analysis of parametric problems, because each parameter of interest can be introduced into the reduced framework. In [2] a rotating electrical machine was linearly modelled with varying material parameters. Another linear parametric problem of an electric machine is presented in [1]. However, it is limited by a structured finite element grid. Electrical machines are constructed of flux guiding material, which exhibits nonlinear behavior. A nonlinear transformer is simulated in [3] with the PGD and a method to reduce the computational effort related to the evaluation of the nonlinear material by employing the Discrete Empirical Interpolation Method (DEIM) is shown. In this paper a permanent magnet excited synchronous machine (PMSM) with a parametric excitation is simulated and analysed while considering the nonlinear material. Furthermore, no restriction to structured meshes is necessary. The current excitation of rotating electrical machines is commonly denoted in the dq-plane. To enable the PGD to approximate the machine behavior in different operating points and for different permanent magnet materials, the method is extended by parameters for the permanent magnet remanence B_{r} as well as the current amplitude $\hat{\mathrm{I}}$ and current angle ϕ.

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II. NONLINEAR SIMULATION OF ELECTRICAL MACHINES

Due to the complicated geometry of electrical machines the Finite Element Method is used, with the magnetic vector potential. Due to the non-linear behavior of electrical steel, it is necessary to employ a nonlinear iteration scheme, such as the fixed-point method

$$\begin{aligned}&\int_\Omega \nu_{\mathrm{fp}} \nabla\times A(x,\theta,\dots)\nabla\times A^*(x,\theta,\dots)\,\mathrm{d}\Omega\\ &=\int_\Omega J(x,\theta,\dots)A^* - \nabla\times \mathrm{A}^*\, H_{\mathrm{fp}}\big(A(x,\theta,\dots)\big)\,\mathrm{d}\Omega,\end{aligned} \tag{1}$$

where A is the vector potential, J is the excitation vector, x denotes the space, θ the angle between the line voltage V_L and the synchronous generated internal voltage V_P and ν_{fp} is the fixed-point reluctivity. The nonlinearity is considered by the virtual magnetization vector H_{fp}.

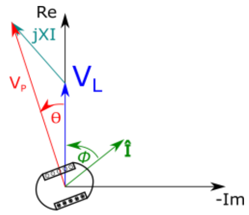

Fig.1: Current and angle definitions.

To exactly characterize the machine behavior in a defined speed range various operating points have to be simulated. These points are usually denoted in the dq-frame (Fig. 1). The mechanical angular position of the rotor is given by $\theta - 90°$. If further the remanence B_{r} is a variable design parameter, the number of simulations to be conducted is determined by the parameter combinations:

$$N_{\mathrm{Sim}} = \theta \cdot \hat{\mathrm{I}} \cdot \phi \cdot B_{\mathrm{r}}. \tag{2}$$

III. PARAMETRIC PROPER GENERALIZED DECOMPOSITION

The Proper Generalized Decomposition approximates the magnetic vector potential $\vec{A}$ by a separated form. Each parameter x and p is represented by an own functional term $R_i(x)$ and $F_{n,\mathrm{i}}(p_n)$:

$$A(x,p_1,\dots,p_{\mathrm{n}}) \approx \sum_{\mathrm{i=1}}^{\mathbf{m}} \mathrm{R_i}(x)\cdot F_{1,\mathrm{i}}(p_1)\cdot \dots \cdot F_{\mathrm{n,i}}(p_{\mathrm{n}}) \tag{3}$$

Approximation (3) is introduced into the magnetic vector potential formulation (2) and a consecutive alternative direction scheme is employed to solve for the parameter function of the decomposition (4).

$$
\begin{aligned}
&\sum_{\mathrm{i}} \int_{p_1} F_{1,\mathrm{i}}(p_1) F_{1,\mathrm{m}}(p_1)\,\mathrm{d}p_1 \cdot \ldots \\
&\cdot \int_{p_\mathrm{n}} F_{\mathrm{n,i}}(p_\mathrm{n}) F_{\mathrm{n,m}}(p_\mathrm{n})\,\mathrm{d}p_\mathrm{n} \int_{\Omega} \nu_{\mathrm{fp}} \nabla \times R_{\mathrm{i}}(x) \nabla \\
&\qquad\qquad \times R_{\mathrm{m}}(x)\,\mathrm{d}\Omega \\
&= \int_{p_1} J_{1,i}(p_1) F_{1,m}(p_1)\,\mathrm{d}p_1 \cdot \ldots \\
&\cdot \int_{p_n} J_{n,i}(p_n) F_{n,m}(p_1)\,\mathrm{d}p_n \int_{\Omega} J(x) R_m(x)\,\mathrm{d}\Omega \\
&- \int_{p_1} F_{1,m}(p_1) \cdot \ldots \\
&\cdot \int_{p_n} F_{n,m}(p_n)\,\mathrm{d}p_n \int_{\Omega} H_{\mathrm{fp}}\big(A(x,\theta,\ldots)\big)\,\mathrm{d}\Omega \mathrm{d}p_\mathrm{n} \\
&\qquad\qquad \cdot \ldots \mathrm{d}p_1
\end{aligned} \tag{4}
$$

It can be seen, that all integrals except for the nonlinearity can be computed in a decomposed way. The nonlinear term H_{fp} depends on the approximation of the solution A. To reduce the computational effort of evaluating H_{fp} in the reference system the DEIM is utilized [3].

IV. Parametric Simulation of a Synchronous Machine

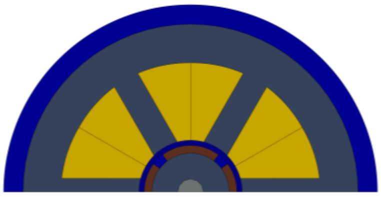

Fig. 2: Simulated Synchronous machine.

The parameters for the following parametric analysis of the given synchronous machine (Fig. 2) are the current angle ϕ with a fixed current amplitude $\hat{I}$, the remanence of magnets B_r as well as the space x. The current angle varies between 0 and 360 degree in 20 steps and the remanence is varied between 0.8 T and 1.2 T in three steps.

$$
\begin{aligned}
&\vec{A}_{PGD}\big(x, \hat{\mathrm{I}}_{\mathrm{dq}}, \phi_{\mathrm{dq}}, B_\mathrm{r}\big) \\
&= \sum_{\mathrm{i}=1}^{\mathrm{m}} R_{\mathrm{i}}(x) \cdot F_{1,\mathrm{i}}(\theta) \cdot F_{2,\mathrm{i}}\big(\hat{\mathrm{I}}_{\mathrm{dq}}\big) \cdot F_{3,\mathrm{i}}\big(\phi_{\mathrm{dq}}\big) \cdot F_{4,\mathrm{i}}(B_\mathrm{r})
\end{aligned} \tag{5}
$$

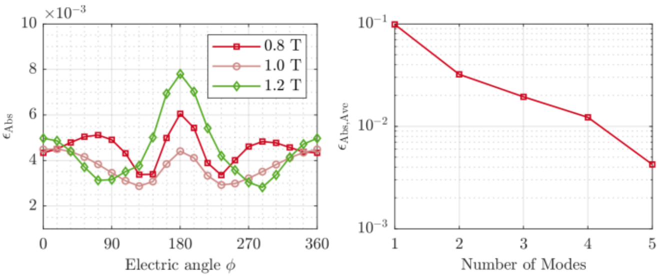

Fig 3a: Mathematical Error for different magnet excitations at different current angles

Fig 3b: Average mathematical versus number of modes

Fig 3: Lockstep simulation for different permanent magnet remanence values.

The absolute residual ϵ_{Abs} for distinct parameter combinations (Fig. 3a) of the remanence and current angle illustrates that the error is smaller than 1 % for each parameter combination. The absolute residual averaged over all parameter combinations:

$$
\epsilon_{\mathrm{Abs,Ave}} = \sum \frac{\|MA_{\mathrm{PGD}} - B\|}{\|B\|} \Big/ N_{\mathrm{Sim}} \tag{6}
$$

drops below 0.5% after 5 modes are enriched into the PGD (Fig. 3b) . Fig. 4 emphasizes that the reduced model approximates the torque characteristic well.

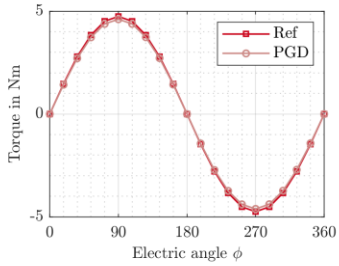

Fig. 4: Torque of locked rotor simulation for 0.8 T.

V. Conclusions

The PGD is employed to simulate a permanent magnet excited synchronous machine with varying excitation parameters such as the remanence flux density of the PMSM and a locked rotor simulation is discussed. The results highlight that the torque can be well approximated and the mathematical error converges towards a reasonable value below 0.5%. In the final paper different operating points in the dq-plane as well as the rotation will be considered. The influence of the number of parameters on the convergence [7] to achieve a technical relevant accuracy will be studied.

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