Error Estimators for Proper Generalized Decomposition (PGD) in Time Dependent Electromagnetic Field Problems

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Due to fine discretization in space and time, the simulation of transient electromagnetic phenomena results in a large system of equations. To cope with this computational effort model order reduction techniques can be employed. To assess the accuracy of the solution of the reduced model, an error estimation is crucial. A commonly used approach consists of the evaluation of the deviation between the reduced and the full model. This yields a loss of the a-priori property of the proper generalized decomposition. To overcome this problem two a-priori criteria are presented in this paper.

Index Terms—**Error criteria, finite element method, model order reduction, proper generalized decomposition**

I. Introduction

Large scale finite element models arise from e.g. time dependent electromagnetic field problems, due to the skin depths of the eddy currents. On the one hand, to properly model eddy currents, the conducting regions have to be accurately discretized in space. On the other hand the time interval has to be accurately sampled to consider all transient effects. The resulting computational effort of these transient simulations can be reduced by model order reduction (MOR). The reduction techniques can be distinguished in two classes, namely a-posteriori and a-priori methods. One well known a-posteriori method is the proper orthogonalized decomposition (POD), which is based on collecting snapshots of the reference system to calculate a reduced representation. A-priori methods such as the proper generalized decomposition (PGD) method construct a reduced order model without any previously obtained solutions [1]. While different error criteria for a-posteriori methods have been formulated [4], a reasonable criterium for a-priori methods is not yet stated. To maintain the a-priori property of the PGD, an a-priori error criterium is presented in the following.

II. Magnetoquasistatic Problem

To solve the magnetoquasistatic field problem the Finite Element Method (FEM) with the magnetic vector potential $\boldsymbol{A}$ is employed (1). The problem consists of a domain with unary boundary conditions and a conducting subdomain, which allows eddy currents.

$$\nabla \times \nu(\nabla \times \boldsymbol{A}(t)) + \frac{\sigma \mathrm{d}\boldsymbol{A}(t)}{\mathrm{d}t} = \boldsymbol{J}(t) \tag{1}$$

III. Proper Generalized Decomposition

The basic principle of the PGD is to decompose the solution of a linear partial differential equation (PDE) into a sum of m tensor products [2] (2).

$$\boldsymbol{A}(\mathrm{x},\mathrm{t}) \approx \sum_{i=1}^{m} \boldsymbol{R}_i(x) S_i(t) \tag{2}$$

An alternative direction scheme is adaptated to enrich the PGD basis, based on fixing one component while solving the other one [3].

To further improve the decomposition an update step can be applied, which introduces an additional step after a new pair $\boldsymbol{R}(x)$ and $S(t)$ has been found. The update is applied by using an operator $W(\mathbf{R}(x))$, which maps all space modes into the time domain and solves all time functions $S(t)$ at once [1].

IV. Accuracy of the PGD

Even though the PGD is applied to many areas, the error evaluation and the information content of the single modes were not a main focus of research here. The enrichment is terminated after a certain a-posteriori relative error is fulfilled or until a defined number of modes are enriched [3,5-7]. To overcome this disadvantage different error criteria are introduced and compared in this paper.

A. A-Posteriori Error Criteria

The need for a reference solution, which has to be obtained from the complete system of equations, characterize a-posteriori error criteria. Common criteria in this context use the magnetic energy (3), the Joule losses (4) or the reference solution $\mathbf{X_{ref}}$ (5) evaluated using the two norm.

$$\epsilon_{mag} = \frac{||W_{\mathrm{mag,ref}} - W_{\mathrm{mag,PGD}}||_2}{||W_{\mathrm{mag,ref}}||_2}, \tag{3}$$

$$\epsilon_j = \frac{||P_{j,ref} - P_{\mathrm{j,PGD}}||_2}{||P_{\mathrm{j,ref}}||_2}, \tag{4}$$

$$\epsilon_{LSQ} = \frac{||X_{PGD} - X_{ref}||_2}{||X_{ref}||_2} \tag{5}$$

B. A-priori Error Criteria

To retain the a-priori property of the PGD, two criteria are presented in the following paragraphs. Combining these two leads to a reasonable measure of relative and absolute convergence of the decomposition.

1) Absolute Residual

Instead of comparing the reference solution to the PGD solution it is more convenient to compute the absolute residual (6). Although a reference solution is not required, the evaluation of all time steps in (6) with the reference system matrix $\mathbf{M}$ and the time dependent excitation $\boldsymbol{J}(t)$ is still necessary, resulting in high computational efforts. This criterium can be interpreted as an a-priori version of (5) and yields the absolute residual.

$$\epsilon_{Abs} = \frac{||\mathbf{M}\boldsymbol{X}_{\text{PGD}}(t) - \boldsymbol{J}(t)||_2}{||\boldsymbol{J}(t)||_2} \tag{6}$$

2) Information content

Another approach can be formulated by using the singular value decomposition. Under the assumption, that the singular values of the system decrease rapidly, they can be used as a measure of convergence of the enrichment. The evaluation of the PGD solution in a certain time step can be reformulated into matrix form by

$$\begin{aligned} \boldsymbol{A}(x,t) &\approx \sum_{i=1}^{m} \boldsymbol{R}_i(x) S_i(t) \\ &= \mathbf{M}_{\text{R}} \cdot \boldsymbol{S} \ . \end{aligned} \tag{7}$$

In (7) $\mathbf{M}_{\text{R}}$ is a matrix with the space modes $\boldsymbol{R_i}$ as columns and $\boldsymbol{S}$ is a vector with the values of $S_i(t)$ in the evaluation timestep as entries. The matrix $\mathbf{M}_{\text{R}}$ can be decomposed by a singular value decomposition and the resulting singular values give a hint of the information content of the modes, because $\mathbf{M}_{\text{R}}$ acts as a linear projection on $\boldsymbol{S}$.

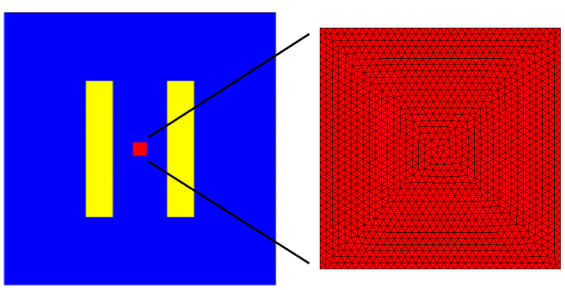

Fig. 1. Academic example of a magnetizing coil (yellow) with a conductive sample (red).

C. Application

The previously discussed criteria are applied to an academic example of a coil with a conductive sample inside (s. Fig. 1). The conductivity of the square sample with 10 mm edge length is equal to 10 kS/m. The coil is operated by a sinusoidal current of 100 A and frequency of 1 kHz. The relative permeability of the sample is arbitrary set to 2183.2 and one period was divided into 250 equidistant steps. From Fig. 2 it can be depicted that the eddy current loss of the reduced order model (ROM) are in good agreement with the reference solution, the deviation is smaller than 0.15% for 8 modes. Fig. 3 shows the singular values of $\mathbf{M}_{\text{R}}$ (7) and it can be recognized, that the singular values decrease fast after the 5th.

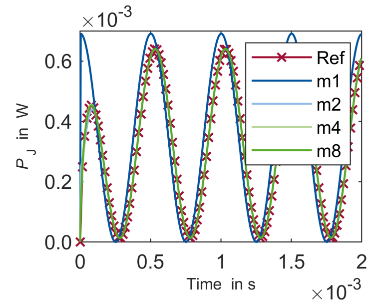

Fig. 2. Eddy Current Losses.

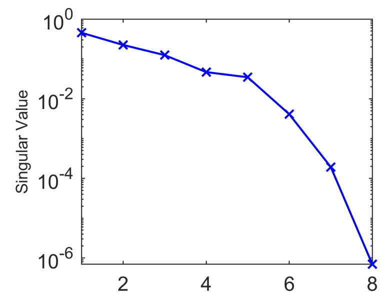

Fig. 3. Singular Values of $\mathbf{M}_{\text{R}}$ for 8 Modes.

V. Conclusions

A measure for convergence of the enrichment process of the PGD has been proposed, which is not based on reference solutions nor on the reference system. In combination with (6) the a-priori property of the PGD can be kept, while getting important information of the relative and absolute convergence.

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