Anisotropic Model for Villari Effect in Non-Oriented Electrical Steel Sheets

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***Abstract*—An energy-based approach for anisotropic modeling of stress-dependent magnetization curves in electrical steel sheets is presented. The model is based on an orthotropic extension of an existing isotropic thermodynamic model. The model fits well to experimental results.**

***Index Terms*—Hysteresis, magnetic materials, magnetostriction, stress, Villari effect.**

I. Introduction and Methods

Magnetic behavior of electrical steel sheets under mechanical stress is a rather complicated problem due its intrinsic three-dimensionality. Recent development in magneto-mechanical modeling starts to allow accurately accounting for arbitrary orientations between the magnetic field and the stress [1]-[3], but so far only isotropic models have been presented. In this paper, we describe an approach for modeling the Villari effect with orthotropic anisotropy.

An isotropic magneto-elastic model is derived first. We are looking for a thermodynamic free energy as a function of the magnetic flux density and mechanical stress: $\varphi_{\mathrm{me}}(\boldsymbol{B},\boldsymbol{\sigma})$, from which we can derive the magnetic field strength as

$$\boldsymbol{H}(\boldsymbol{B},\boldsymbol{\sigma}) = -\frac{\partial \varphi_{\mathrm{me}}(\boldsymbol{B},\boldsymbol{\sigma})}{\partial \boldsymbol{B}} \tag{1}$$

In an isotropic case the energy can only depend on invariants $I_1 = \mathrm{tr}\,\boldsymbol{\sigma}$, $I_2 = \mathrm{tr}\,\boldsymbol{\sigma}^2$, $I_3 = \det \boldsymbol{\sigma}$, $I_4 = \boldsymbol{B}\cdot\boldsymbol{B}$, $I_5 = \boldsymbol{B}\cdot(\boldsymbol{sB})$, and $I_6 = \boldsymbol{B}\cdot(\boldsymbol{s}^2\boldsymbol{B})$, where $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}(\mathrm{tr}\,\boldsymbol{\sigma})\boldsymbol{I}$ is the deviatoric part of the stress. I_1–I_3 do not affect the field strength and are thus not considered here. We express φ_{me} as a tricubic spline of $I_{4\ldots6}$, which are divided into M_4, M_5 and M_6 intervals, respectively, so that the energy becomes piecewise defined in $M = M_4M_5M_6$ cubes. To index these cubes, a linear index $m = 1, \ldots, M$ is used. Equation (1) now becomes

$$\boldsymbol{H} = -\sum_{m=1}^{M}\sum_{i=0}^{3}\sum_{j=0}^{3}\sum_{k=0}^{3} c_{m,ijk}\left(iI_4^{i-1}I_5^{j}I_6^{k}\frac{dI_4}{d\boldsymbol{B}} + jI_4^{i}I_5^{j-1}I_6^{k}\frac{dI_5}{d\boldsymbol{B}} + \ldots \right.$$
$$\left. \ldots + kI_4^{i}I_5^{j}I_6^{k-1}\frac{dI_6}{d\boldsymbol{B}} \right) \tag{2}$$

in which the spline coefficients $c_{m,ijk}$ are nonzero only in cube m and zero elsewhere. They can be identified from measured magnetization curves by linear least-squares fitting.

The isotropic model can be extended to an orthotropic case by replacing the scalar coefficients $c_{m,ijk}$ with tensors

$$\boldsymbol{c}_{m,ijk} = c_{1,m,ijk}\boldsymbol{E}_1 + c_{2,m,ijk}\boldsymbol{E}_2 + c_{3,m,ijk}\boldsymbol{E}_3, \tag{3}$$

where $\boldsymbol{E}_n = \boldsymbol{e}_n \otimes \boldsymbol{e}_n$ and $c_{n,m,ijk}$, $n = 1, 2, 3$, respectively, are the structural tensors corresponding to the orthotropic symmetry group and the coefficients of their linear combination. Vectors $\boldsymbol{e}_n$ are the mutually orthogonal unit vectors and $\otimes$ designates the tensor product. In orthotropic case, $\boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3 = \boldsymbol{I}$.

II. Results and Discussion

Fig. 1 shows results of fitting the model to magnetization curves measured under uniaxial tensile stresses from non-oriented electrical steel sheets both in the rolling (RD) and transverse directions (TD). In the RD, the stress causes the permeability to decrease monotonically, while in the TD, small tensile stress increases the permeability. This behavior is modeled correctly with the presented model. In the full paper, we will discuss properties of the model in more details and consider also hysteresis.

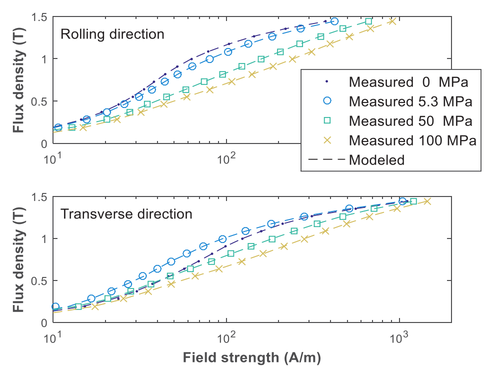

Fig. 1. Comparison of the anisotropic model and magnetization curve measurements under tensile stresses in rolling and transverse directions

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