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A simplified model of ferromagnetic sheets considering the magnetization dynamics utilizing the saturation wave model

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The usual skin effect observed in magnetically linear medium ($\mu = \text{const.}$) is absent in a magnetically non-linear medium, leading to wrong predictions of the eddy current field using the classical approach. For this reason, this paper proposes a thin sheet model, improving the eddy current field description on the basis of physical ideas in the framework of the saturation wave model, which describes the dynamic magnetization of the material with rectangular hysteresis loop. Therewith, the layer-to-layer nature of the magnetization reversal is taken into account. The hysteresis is modeled by means of a static history-dependent hysteresis model. This leads to a simplified model of conducting ferromagnetic sheet, which describes magnetization of isotropic electrical steels. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4861682]

I. INTRODUCTION

The accurate prediction of iron losses and magnetization behavior of electrical steel sheets for various frequencies and magnetic flux densities taking into account magnetic hysteresis, induced eddy currents, and so-called excess (anomalous) loss is eminent for an accurate design of electrical machines and other devices containing ferromagnetic cores.

The ideal solution would be a model, which allows predicting both, the specific loss and the shape of hysteresis loop at arbitrary magnetization regimes.

Utilizing a magneto-dynamic model (MDM),[1] which is a finite-difference (FD) or finite-element (FE) solver of the classical Maxwell (penetration or diffusion) equation (1), a sufficiently accurate description of transients in the laminated non-oriented (NO) steel can be obtained

$$\rho \frac{\partial^2 \vec{H}}{\partial x^2} = \frac{\partial \vec{B}}{\partial t}, \tag{1}$$

where vectors of the magnetic field strength $H(x, t)$ and magnetic flux density $B(x, t)$ are directed along the sheet; axis x is normal to its surface, and ρ is the specific electrical resistivity of the steel. The magnetic flux density $B_k(t)$ and magnetic field $H_k(t)$ in a node k of the computational grid are linked by a static hysteresis dependence $H_h(B)$, in which the excess loss caused by the domain structure is reproduced by a time delay of $B_k(t)$ behind $H_k(t)$.

The solution of the partial derivative equation (1) can be reduced to the integration of N simultaneous ordinary differential equations for $B_k(t)$ and $H_k(t)$. Sufficient accuracy is obtained using 15 to 25 nodes in the FD- or FE-grid. However, the dimension of the problem increases with the number of branches, when analyzing devices with branched magnetic topology. This leads to a complicated and time-consuming model. Substantial simplification of the problem, while keeping sufficiently accurate solution, can be achieved by using a thin sheet model (TSM), which links the magnetic field at the sheet surface $H(t)$ and the mean magnetic flux density $B(t)$ over its cross section.[2,3] The magnetic field $H(t)$ in such a model is represented by the sum of hysteresis, induced eddy-current (classical) and excess components, $H(t) = H_h + H_{clas} + H_{exc}$. For example, the TSM in Refs. 2 and 3 is written by the equation

$$H(t) = H_h(B) + \frac{d^2}{12\rho}\frac{dB}{dt} + g(B)\delta\left|\frac{dB}{dt}\right|^{1/\alpha}, \tag{2}$$

where d is the sheet thickness and $\delta = \text{sign}(dB/dt)$. Among the advantages of (2) relative to other TSMs is its possibility to change frequency properties of the model (by choosing α) and to control the shape of dynamic hysteresis loops by choosing function $g(B)$.

Model (2) has shown considerable accuracy being applied to grain-oriented[3] and some NO steels (commonly assumed to have isotropic magnetic properties[4] and typically employed in rotating electrical machines) with sheet thickness 0.1 mm and high silicon content (5.5% and 6.5%).[3] At the same time, this model is less accurate in describing high-frequency regimes of conventional NO steels, i.e., steels with the sheet thickness about 0.5 mm and silicon content not exceeding 3%. One reason is the relatively low electrical resistivity of these steels and, as a result, the error of the well-known formula for the so-called classical field[4]

$$H_{clas}(B) = \frac{d^2}{12\rho}\frac{dB}{dt}. \tag{3}$$

It should be recalled that (3) is valid, strictly speaking, for a linear dependence $B(H)$ and for negligibly small derivatives dB/dt, i.e., at low frequencies f. The error of (3) at elevated frequencies is illustrated in Fig. 1 where curve

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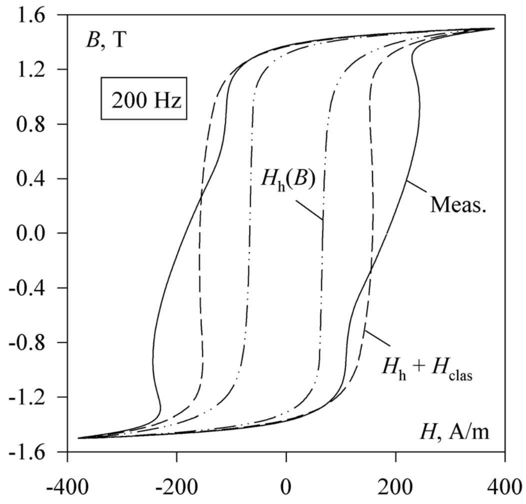

FIG. 1. The first (H_h) and two first terms ($H_h + H_{clas}$) in (2) compared to a measured loop (straight line).

$H_h + H_{clas}$ was constructed by summation of the first and second terms in (2). It is seen that this curve goes outside the experimental dynamic loop measured for NO steel with $d = 0.5$ mm and $\rho = 0.43 \times 10^{-6}$ Ωm in an Epstein frame with controlled sinusoidal magnetic flux density. This means that the third term in (2) should be negative along the lower segment of the ascending branch (and, respectively, along the upper segment of the descending branch) that contradicts the physical meaning of the field component. In this connection, the TSM was artificially modified in such a way as to make the eddy-current component in (2) dependent not only on derivative dB/dt but also on the magnetic flux density $B(t)$[5]

$$H(t) = H_h(B) + \frac{d^2}{12\rho}\delta\left|\frac{dB}{dt}\right|^{\gamma(B)} + g_0\delta\left|\frac{dB}{dt}\right|^{1/2}. \qquad (4)$$

The power γ in (4) was determined in Ref. 5 by the expression $\gamma(B) = a_1 + a_2\delta B + a_3 B^2$, where coefficients a_1, a_2, a_3, and multiplier g_0 were found by means of an iterative technique, in which the loops calculated at peak value B_p (mainly at $B_p = 1.5$ T) and 2–3 frequencies were fitted to correspondent measured loops. The study of model (4) shows that it is mainly acceptable at the flux density employed in the iterative fitting procedure, i.e., at $B_p = 1.5$ T. As B_p decreases, the accuracy of (4) also quickly decreases. Thus, for example, at $B_p = 1$ T and $B_p = 0.5$ T the error in the specific loss calculation increases to 25% and 50%, respectively. For this reason, this paper proposes a TSM where the eddy current field description is improved on the basis of physical ideas yielding to a model, which is sufficiently accurate outside the flux density and frequency range used for parameter identification.

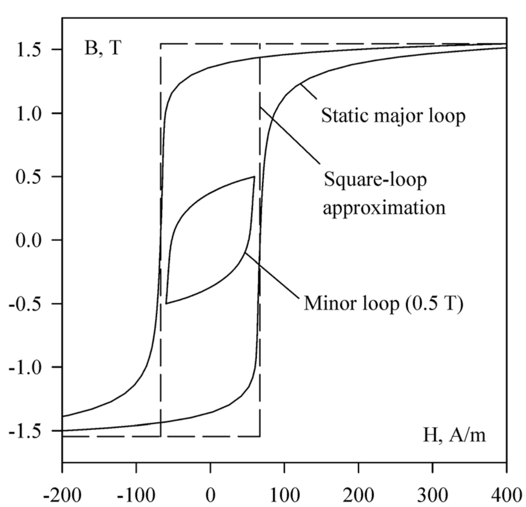

FIG. 2. Static loops of NO steel at 1.5 T and 0.5 T as well as a corresponding RHL approximation of the major loop (dashed lines).

II. IMPROVED THIN SHEET MODEL

Fig. 2 shows the static major loop of the studied NO steel as well as a minor symmetrical loop measured at $B_p = 0.5$ T. It is apparent that their square-loop approximations (dashed lines in Fig. 2) are more accurate than any linear approximations of these loops (especially major loop). This raises the idea that a better description of the magnetization dynamics can be carried out in the framework of the saturation wave model (SWM) originally proposed by Wolman and Kaden[6] and then further developed in several works.[7,8] The SWM applies to a ferromagnetic material characterized by a steplike magnetization curve with maximum value B_{max}. As a consequence, the model can be applied to a material with a rectangular hysteresis loop (RHL) with the height $2B_{max}$. In accordance with the SWM, the magnetization reversal in the RHL material has a layer-to-layer nature and consists of successions of instant flux reversals in thin layers of the sheet (from $-B_{max}$ to $+B_{max}$ and back). The potential applicability of the SWM in the TSM is corroborated by the layerwise flux reversal (Fig. 3) calculated by means of accurate, but computationally expensive MDM[1] at sinusoidal average flux density with amplitude $B_p = 1.5$ T and frequencies 50 and 400 Hz.

Similarly to the process in the material with ideally square loop, maximum flux density in all layers of the sheet, including magnetic flux density B_{surf} at its surface and the magnetic flux density B_{mid} in the middle of the sheet, reaches the level of $B_p = 1.5$ T, i.e., the usual skin effect observed in a magnetically linear medium is absent here. Therefore, the second term in (2) is replaced by the expression arising from the SWM

$$H_{ec}(B) = \frac{d^2(B - B_T)}{8\rho B_{max}}\frac{dB}{dt}. \qquad (5)$$

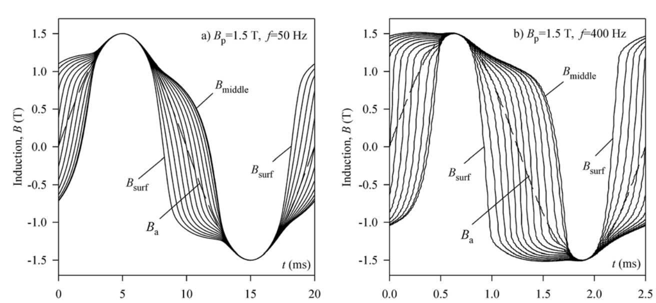

FIG. 3. Flux densities at equidistant points from the surface to the middle of the sheet (solid curves). Dashed curve is the sinusoidal average magnetic flux density B_a versus time.

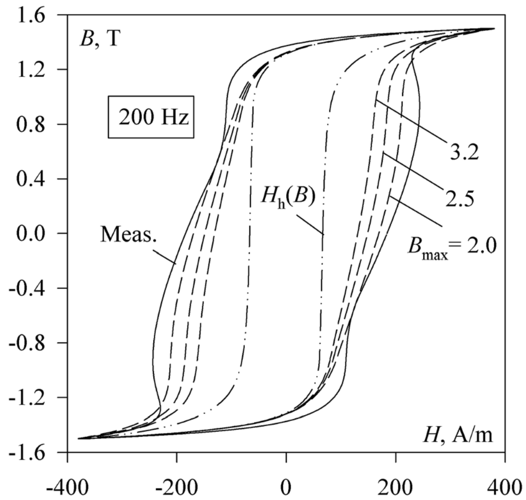

FIG. 4. Sum of $H_h(B)$ and $H_{ec}(B)$ calculated for $B_{max} = 2.0$, 2.5, and 3.2 T.

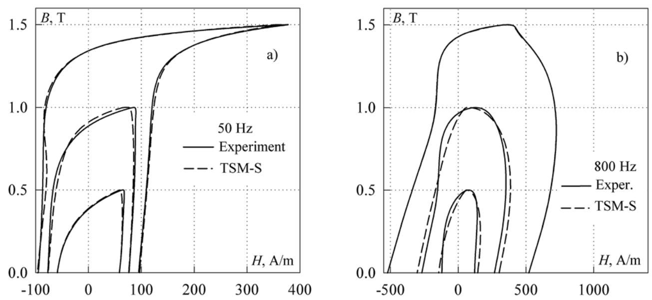

FIG. 5. Predicted and measured dynamical hysteresis loops at 50 Hz (left) and 800 Hz (right).

Here, B_T being the magnetic flux density in a turning point, which is the point where derivative dB/dt changes its sign. Therewith, the step-by-step character of magnetic flux density changes in the sheet under arbitrary magnetic flux density waveforms is accounted for. Dashed curves in Fig. 4 show the sum of the hysteresis field $H_h(B)$ and the eddy-current field $H_{ec}(B)$ calculated with (5) for several values of B_{max} (2.0, 2.5, and 3.2 T). As these curves do not (or almost do not) go beyond the experimental loop, the field H_{exc} supplementing this sum to the measured field $H(t)$, is positive at any $B(t)$.

So, the proposed phenomenological model, referred to as TSM-S, is written as

$$H(t) = H_h(B) + \frac{d^2(B - B_T)}{8\rho B_{max}}\frac{dB}{dt} + g(B)\delta\left|\frac{dB}{dt}\right|^{1/\alpha(B)}. \quad (6)$$

Since the static hysteresis loop of the real magnetic material is not perfectly square, the value of B_{max} can be considered a variable phenomenological parameter, which is chosen for a given material along with functions $\alpha(B)$ and $g(B)$. In order to identify the fitting parameters, at first, the required $g(B)$ and $\alpha(B)$ are configured for a chosen value of B_{max}, so as the loops calculated for a fixed amplitude B_p were as close as possible to the loops measured at 3–4 frequencies (ensuring a stable and well-conditioned fitting process). Given the symmetry of the steady-state hysteresis loops, the model-fitting can be made using n points of the ascending branches. For each such point (i.e., for a given level B_i), a pair of α_i and g_i is found so as to minimize the total deviation in H of calculated loops from experimental ones for all chosen frequencies. The discrete results from above calculations can be approximated by splines $\alpha(B)$ and $g(B)$. The calculations for the chosen B_{max} are completed by building frequency dependences of the specific iron losses for several values of B_p. After comparing calculated dependencies $W(f)$ with experimental ones, the value of B_{max} is corrected and the above procedure is repeated until a best fit of calculated and experimental curves $W(f)$ is achieved.

III. MODEL VERIFICATION

The fitting of model (6) was carried out by using three dynamic loops with $B_p = 1.5$ T, taken at 50, 200, and 800 Hz. The best value of B_{max} was found to be equal 2.9 T. In Fig. 5, calculated dynamic loops are compared with corresponding loops measured at sinusoidal induction. As seen in Fig. 5, the experimental loops used for model fitting, i.e., the loops at $B_p = 1.5$ T, are reproduced by the model almost exactly. The prediction of the loop shapes at lower B_p is satisfactory.

IV. CONCLUSIONS

The motivation for this work is the development of a time-efficient model of conducting ferromagnetic sheet, which describes magnetization of isotropic electrical steels considering magnetic hysteresis, induced eddy currents as well as excess (anomalous) loss. Sufficiently accurate solutions can be obtained using a TSM. Improvement of the TSM proposed is achieved by determining eddy current component of the magnetic field at the sheet surface in the framework of the method by Wolman and Kaden, which describes the dynamic magnetization of the material with rectangular hysteresis loop. Thereby, the layer-to-layer nature of the magnetization process is taken into account. The effectiveness of the proposed algorithm is confirmed by modeling a NO electrical steel with d = 0.5 mm and $\rho = 0.43 \times 10^{-6}$ Ωm characterized in an Epstein frame with controlled sinusoidal magnetic flux density. Furthermore, due to its physical identity, the model is capable of satisfactorily predicting hysteresis loops under arbitrary magnetization regimes.

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