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ANALYSIS OF FINITE ELEMENT MESH DISCRETIZATION FOR FASTER AND MORE ACCURATE PROBABILISTIC SIMULATIONS: APPLIED TO COGGING TORQUE CALCULATIONS IN ELECTRICAL MACHINES

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Abstract ***– The consideration of uncertainties in the numerical computation of electromagnetic fields has recently gained a lot of attention [11]. Most publications focus on the creation of models for the uncertainty propagation, however neglect the inaccuracy respectively uncertainty of the applied finite element model itself. Mesh quality is one of the parameters determining the numerical model's accuracy. Hence, this paper analyses the influence of mesh accuracy at the example of stochastic cogging torque variations, which are caused by magnetization uncertainties in a permanent-magnet synchronous-machine. As a result, a method to calculate improved probability predictions at minimum computational cost is presented and applied here.***

Introduction

During the production process of electrical machines, parameters as intrinsic deviations of the soft- and hard-magnetic materials due to the manufacturing process result in small variations of each machine's behavior. These variations are considered as uncertainties and need to be included into the machine's simulations. For this purpose, recent publications propose polynomial-chaos (PC) decompositions spanned by random polynomials as in [1].

PC-decompositions aim at representing the uncertain data as polynomial functions in dependence of a set of independent and orthogonal random variables with known probabilistic densities. Subsequently, statistical analysis can be performed by post-processing the coefficients of the determined polynomial-chaos meta-model. Two different approaches exist to identify the PC meta-model: On the one hand, the applied finite element (FE) solver can be recoded to include the stochastic uncertainties as an additional dimension, yielding a reformulated deterministic problem for the stochastic modes and resulting in a so-called intrusive stochastic Galerkin projection [9]. On the other hand, one can solve a series of deterministic problems, which sample the stochastic space. Afterwards this sample set is used to calculate the PC-coefficients; this approach commonly is referred to as non-intrusive stochastic collocation [10]. The authors of [2] offer a comparison of both approaches, concluding that the implementation of intrusive methods is tedious and error-prone while they do not yield any significant advantage in comparison to the non-intrusive, sampling based approaches.

Considering the non-intrusive, sampling based PC-approaches, most publications as for instance [5], focus on the selection of the best set of base polynomials and input samples in order to receive the most accurate polynomial description at the lowest computational cost. The fact, that all those meta-models rely upon FE-samples and hence intrinsically contain discretization errors, which distort the predictions of further calculations, often is neglected. One of the few publications to consider this problem is [3]. The therein-proposed approach however relies on a dual potential formulation, which may not always be at hand, and requires a complete PC decomposition. A different solution approach towards discretization error estimation in combination with PC-expansion is given in [6], which quantifies the solution’s overall error, but does not allow separating the PC approximation influence from the discretization influence in the calculated error measure.

In order to avoid the described mixture of different error sources, we focus on Monte-Carlo simulation (MCS) instead of PC analysis. The aim is an analysis of the influence of discretization errors on stochastic cogging torque simulations, since cogging torque has proven to be especially sensitive to the FEM’s mesh density [4]. Simulations are executed for a permanent-magnet synchronous-machine (PMSM) and employ magnet variations described in [7] as stochastic input source. As a result, a variance-based correction of the cumulative density function (CDF) is proposed and applied, providing a time effective estimation of more accurate CDFs at minimal costs.

Methodology

a) **Conventional** torque calculation:

Different force calculations for electrical machines are state of the art, all of them being derivations of the principal of virtual work as shown in [4]:

$$T_{\varphi} = \left. \frac{\partial W_{co}}{\partial \varphi} \right|_{i=const} \quad (1)$$

with

$$W_{co} = \int_{\Omega} \int_{0}^{\vec{H}} \vec{B} \cdot d\vec{H} d\Omega \quad (2)$$

T_{φ} being the torque, W_{co} the co-energy, B and H symbolizing magnetic flux respectively field in the domain Ω. Equation (2) illustrates, that the prediction quality of torque calculations directly depends on the accuracy of the calculated air-gap’s magnetic field and flux. Improving the air gap’s discretization by adding more element layers yields a convergent behavior of the calculated cogging torques. Figures 1 and 2 picture this behavior for the chosen application:

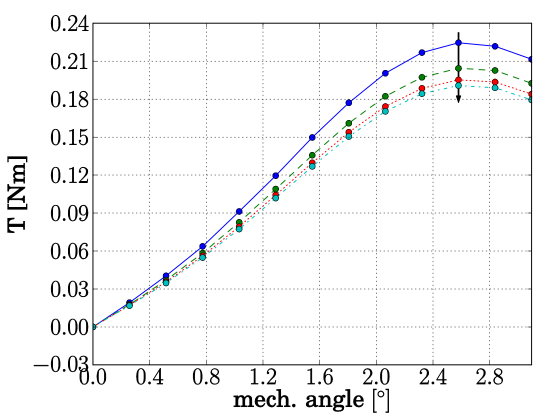

Fig. 1 First quarter of one cogging torque period simulated with an increasing number of air gap layers along the indicated arrow.

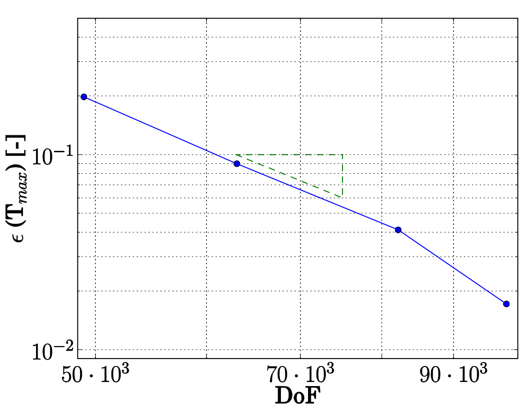

Fig. 2 Error of the cogging torque at angle 2.6º versus the system’s degrees of freedom, employing the Maxwell stress tensor method.

Figure 1 shows the first quarter of one cogging torque period depicted over the rotor’s rotation angle. It can be observed, that the cogging torque’s peak value decreases for an increasing number of air gap layers. [8] offers a detailed analysis of the error’s convergence, also stating that the global calculation error ε can be quantified as a function of the convergence constant C, the element size h and the shape function’s order q:

$$\left\| \pounds \right\| \leq C \cdot h^{q+1} \tag{3}$$

This relationship is confirmed in Figure 2, which pictures the calculation error of the cogging torque's peak value for a fixed rotation angle (2.6º) over the FE-matrix degrees of freedom (DoF), employing the Maxwell stress tensor method. The results of the calculated torque quantities display in accordance with equation (3) the expected, linear decay of the calculation error when depicted over a logarithmic scale, proving the correctness of the presented simulations.

b) **Stochastic** torque calculation:

Adding the consideration of uncertainties introduces a further dimension to the described torque calculations. In the following, a PMSM whose rotor is excited by six surface mounted magnets is investigated. Each magnet's remanence flux-density B_{rem} shall be allowed to vary uniformly between 1.08 T and 1.18 T with a desired value of 1.13 T, previously applied for the simulations in Figure 1 and Figure 2. Owing to these circumstances, the cogging torque's peak value for a given discretization converts from a single value to a spread of values, where each value is connected to its probability of occurrence. Figure 3 depicts the resulting torque distribution for a Monte-Carlo simulation featuring 2000 simulations, applying the coarsest mesh with $49 \cdot 10^3$ DoF from the previous simulations:

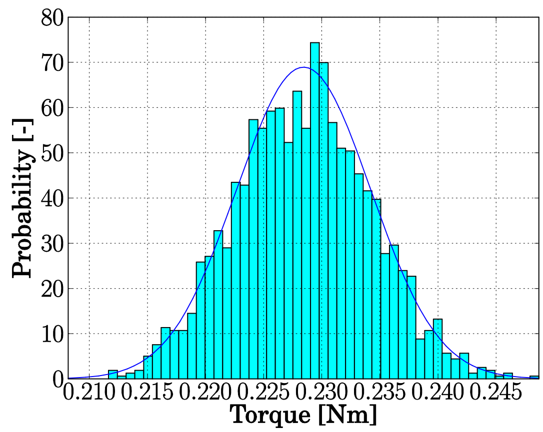

Fig. 3 Histogram and probability distribution of the cogging torque's peak value for six uniformly varying magnets, with simulations based on a coarse mesh having $49 \cdot 10^3$ DoF.

The originating cogging torque peak values in Figure 3 spread in a good agreement to a Gauss distribution, which can be ascribed to the central limit theorem. The distribution's mean value is in accordance with the previous, undisturbed simulation, the occurring high probability values are due to the probability density function's (PDF) standardization to one. In contrast to the seemingly good results, a comparison of the resulting PDF to the known convergence behavior displayed in Figure 2 reveals a severe problem:

- The stochastic calculations are employed in order to improve the prediction quality of the applied FEM simulation. A comparison of the calculated torque's variance to its calculation accuracy however reveals a calculation error which is considerably larger than the calculated variance. Hence, the calculated stochastic prediction is in vain.
- Two conflicting needs have to balanced: On the one hand, introducing a coarser mesh enables a higher number of stochastic variations yielding a more accurate MCS. On the other hand, finer meshes are needed for more accurate torque values.

Results

The presented problem can be overcome by exploiting the knowledge about the torque's error convergence. If one considers the histogram in Figure 3, it is likely that the magnet permutation, which causes the lowest cogging torque, will create one of the lowest cogging torques in a finer discretized simulation as well. Furthermore it is expected, that the relation between the cogging torques of all permutation will stay approximately the same. Accordingly, the probability distribution over all calculated torque values is likely to move between the boundaries of T_{min} and T_{max}. Since the integral over the probability distribution is normalized to one, a shift of the PDF's mean value along with a change of its variance can be predicted. Figure 4 pictures the prediction of the torque's behavior.

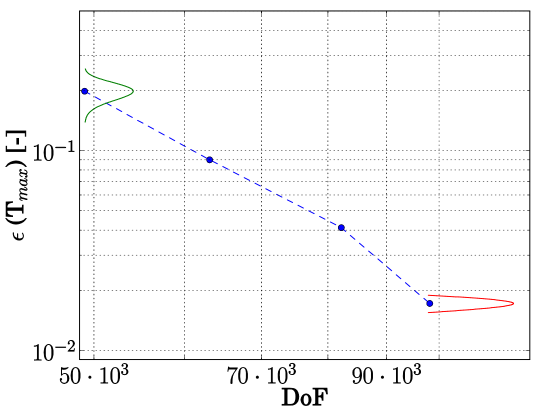

Fig. 4 Prediction of the torque distribution's behavior for an increasingly finer mesh discretization.

The expected behavior has been verified in simulations. Moreover, if the boundary values $^{+}T_{min}$ and $^{+}T_{max}$ for a finer air-gap discretization are known, the coarse torques T(i) may be recast by a mean and variance-based torque approximation to more accurate values $^{*}T$:

$$^{*}T(i) = T(i)\frac{^{+}T_{\max} - {}^{+}T_{\min}}{T_{\max} - T_{\min}} + ({}^{+}T_{\min} - {}^{*}T_{\min}) \quad (4)$$

Figure 5 and displays a comparison of the cogging torque's peak values' cumulative distribution function for a mesh with a coarse (3088 nodes in air-gap, $49 \cdot 10^3$ in total) and a fine (15073 nodes in air-gap, $82 \cdot 10^3$ in total) discretization. Both depicted curves differ completely, showing the disallowable stochastic prediction error made when applying the coarse mesh for stochastic predictions. Figure 6 displays the same curves, subsequent to the application of equation (4). Both cumulative distribution functions now overlap so smooth that only for high zoom levels a difference is apparent. In order to allow an improved analysis of the result's quality, Figure 7 presents a QQ-plot, which depicts the cumulative density function for fine-meshed and coarse-corrected meshed torque calculations. The nearly ideal, linear line confirms the very good agreement of the presented approximation in terms of its probability prediction accuracy.

The presented analysis enables two new approaches: On the one hand, it is possible to use a coarser mesh featuring more stochastic sampling points with subsequent torque correction using (4). This approach yields more accurate stochastic predictions. On the other hand, it is possible to execute an adaptive meshing process, using the boundary values' difference Δ of two consecutive steps as rating criterion with $\Delta(\frac{T_{min}+T_{max}}{2}) < 0.01(T_{min}+T_{max})$ for sufficient good mesh creation in stochastic

applications.

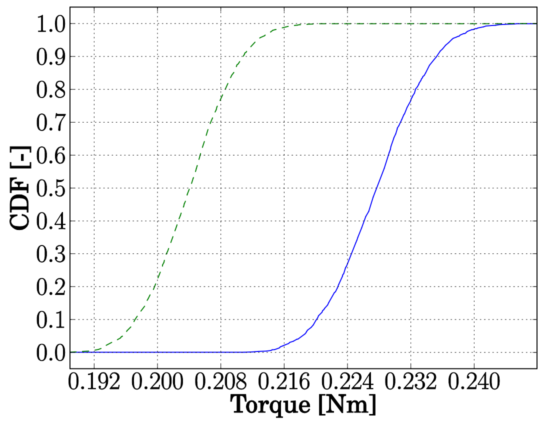

Fig. 5 Cumulative distribution functions (CDF) of cogging torque peak values applying a coarse mesh with $49 \cdot 10^3$ DoF (solid) compared to simulations with a fine mesh featuring $82 \cdot 10^3$ DoF (dashed).

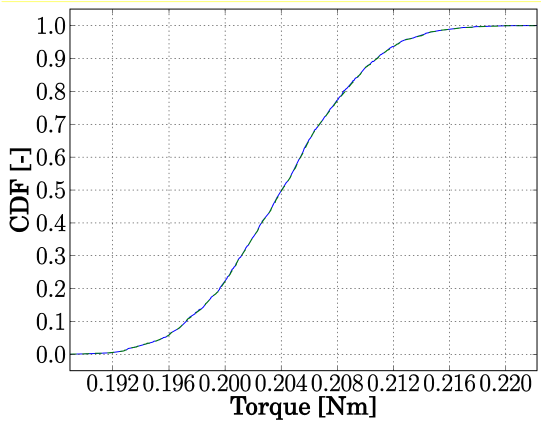

Fig. 6 Cumulative distribution functions (CDF) of cogging torque peak values applying a coarse mesh (solid) with $49 \cdot 10^3$ DoF and corrected using equation (4) in comparison to finer simulations with $82 \cdot 10^3$ DoF (dashed).

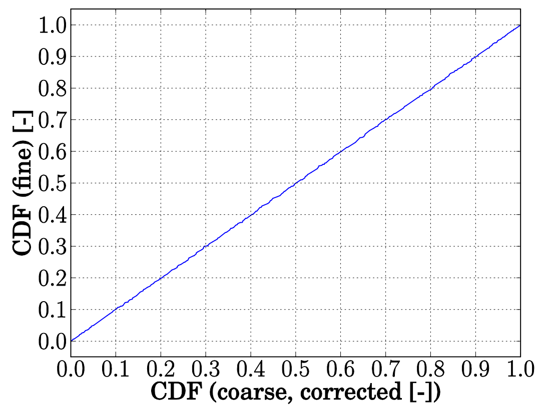

Fig. 7 Quantile-Quantile plot for simulations based on a fine-meshed and coarse, corrected torque calculation. The resulting straight, diagonal line proves the accuracy of the presented correction.

Conclusions

Modeling inaccuracies due to mesh discretization errors introduce additional uncertainties into stochastic simulations, however have been neglected in most recent publications until now. Especially cogging torque is sensitive to these variations. Due to the exponential growth in stochastic simulation costs, complete Monte-Carlo simulations with the needed fine meshes are not always desired. Advanced methods as meta-model based PC analysis do not allow the desired, pure discretization error analysis. Hence, a variance-based evaluation of the cumulative density function's boundaries for different mesh accuracies has been presented. Evaluating the resulting mean and variance values in comparison to the torque's convergence behavior enables to determine the needed mesh quality. If necessary, the proposed correction, which effectively rescales the calculated cumulative density function, can be applied. It offers an improved estimation of the CDF at the cost of only two additional fine simulations. The presented method has been applied to the example of stochastic cogging torque calculations for a six pole PMSM. The results verify the suitability of the proposed approach. Finally, the proposed method also can be employed for correcting PC-sampling points. The resulting implications on the spectral stochastic properties will be researched in future.

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