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A polynomial chaos meta-model for non-linear stochastic magnet variations

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Abstract

Purpose – Due to the production process, arc segment magnets with radial magnetization for surface-mounted permanent-magnet synchronous machines (PMSM) can exhibit a deviation from the intended ideal, radial directed magnetization. In such cases, the resulting air gap field may show spatial variations in angle and absolute value of the flux-density. For this purpose, this paper aims to create and evaluate a stochastic magnet model.

Design/methodology/approach – In this paper, a polynomial chaos meta-model approach, extracted from a finite element model, is compared to a direct sampling approach. Both approaches are evaluated using Monte-Carlo simulation for the calculation of the flux-density above one sole magnet surface.

Findings – The used approach allows representing the flux-density's variations in terms of the magnet's stochastic input variations, which is not possible with pure Monte-Carlo simulation. Furthermore, the resulting polynomial-chaos meta-model can be used to accelerate the calculation of error probabilities for a given limit state function by a factor of ten.

Research limitations/implications – Due to epistemic uncertainty magnet variations are assumed to be purely Gaussian distributed.

Originality/value – The comparison of both approaches verifies the assumption that the polynomial chaos meta-model of the magnets will be applicable for a complete machine simulation.

Keywords Finite element method, Monte Carlo simulation, Manufacturing tolerances, Magnet variations, Stochastics, Magnetic flux-density deviations, Monte Carlo methods, Manufacturing systems, Magnetism

Paper type Research paper

1. Introduction

Many finite element simulations are performed assuming ideal electric machines which feature geometrical and electric symmetric properties. Depending on the number of pole pairs p, the number of slots N and the winding configuration, the air gap field of such an ideal machine shows a spatial periodicity along its circumference. Stochastic deviations of magnets in a permanent-magnet excited rotor, usually created during the magnet production process, strongly can influence machine data such as phase and absolute value of certain torque harmonics in permanent-magnet synchronous machines (PMSM) because the periodic air gap field symmetries are destroyed in these cases.

Typical solutions to handle the mentioned magnet's stochastic variations, introduced by the production, respectively, magnetization process, have been robust machine design

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with worst case estimation and Monte-Carlo simulation as applied in Coenen *et al.* (2011). The drawback of these approaches is that they only allow the calculation of error probabilities by counting result samples which fulfill or fail in the sense of the applied limit state function. Expressing the output size's stochastic deviations in terms of the input size's stochastic variations is not possible. Moreover, Monte-Carlo simulation often requires a large number of simulations for an acceptable error accuracy.

In this paper, a polynomial-chaos meta-model (Clenet *et al.*, 2010; Ramarotafika *et al.*, 2012) is compared to a typical Monte-Carlo simulation to calculate the influence of realistic production deviations onto the magnet's created flux-density. In order to separate field changes caused by stochastic variations in the magnet from field changes caused by interaction with the machine's stator yoke, the field of one sole magnet is simulated and evaluated directly above its surface.

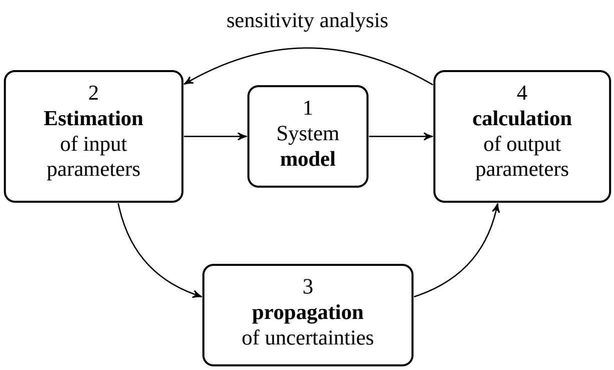
For the simulations, a 2D and a 3D model have been built to compare and estimate the feasibility of applying a 2D magnet simulation. Afterwards, 300 2D simulation have been used for the Monte-Carlo sampling as well as 30 sampling points for the construction of the polynomial meta-model, which then itself has been Monte-Carlo sampled with the same input samples as the original model. The resulting meta-model closely represents the original model as proven in the assessment of the printed cumulative distribution function (CDF), allowing to express the field's stochastics in terms of the input variation stochastics while requiring approximately only one-tenth the sampling size.

2. Methodology

2.1 System model

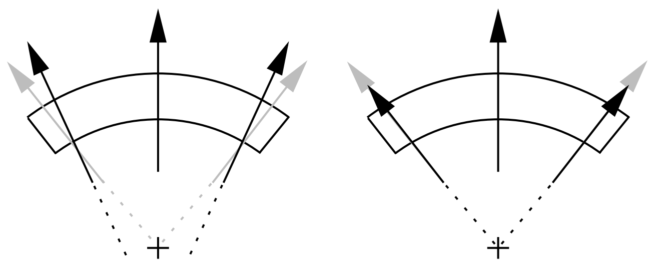
Figure 1 shows the chosen approach for the propagation of the magnet's uncertainties: in step 1, the magnet has been modeled using the finite element method (see Figures 5 and 3 for results). The created models allow two possible error configurations:

(1) Magnetization errors tending from radial magnetization towards an unidirectional magnetization as shown in Figure 2(a). This approach allows for an arbitrary error between both extremes, $\xi_A = 0$ representing complete unidirectional magnetization, $\xi_A = 1$ representing ideal radial magnetization. The resulting magnetic excitation B_{mag} is expressed by equation (1) in dependence of the angle difference to the magnet's center line $\Delta\alpha$ and the radial-to-parallel error factor ξ_A:



Source: Sudret (2007)

Figure 1.
Approach for the uncertainty propagation



Notes: (a) Error type A: deviation of radial magnetization towards unidirectional magnetization; (b) error type B: deviation of local magnetization strength, weakening towards the magnet edge

Figure 2. Considered variations (black) in magnet in contrast to ideal radial magnetization (grey)

$$B_{mag}(\Delta\alpha, \xi_A) = B_r \cdot (cos(\Delta\alpha \cdot \xi_A) \cdot \vec{e}_x + sin(\Delta\alpha \cdot \xi_A) \cdot \vec{e}_y) \quad (1)$$

(2) A spatial changing magnetization remanence magnitude, shaped decreasingly from the magnet center line to the magnet edge as shown in Figure 2(b), $\xi_B = 0$ representing a sinusoidal shaped magnetization remanence, $\xi_B = 1$ representing an ideal uniform value for the magnetization across the entire magnet surface. The resulting magnetization excitation is given in equation (2) in dependence of $\Delta\alpha$ (with $\Delta\alpha$ as in equation (1)) and the angle-attenuation error factor ξ_B:

$$B_{mag}(\Delta\alpha, \xi_B) = (B_r \cdot \xi_B + B_r \cdot (1 - \xi_B) \cdot cos(\Delta\alpha)) \cdot \vec{e}_r \quad (2)$$

After the definition of the error models, a 2D and a 3D FE-model have been built to determine whether 2D simulation would be sufficient and feasible to represent the given magnetization errors. The comparison of the radial flux densities along the evaluation line showed an offset between both models as well as a slightly different scaling in both curves. Removing both effects, the comparison proved both curves to be sufficiently equal in the relevant area (across the magnet surface). Accordingly a 2D model has been chosen for further simulation as it allowed simulation with a finer mesh using the same number of elements. Figures 5 and 3 show the FE-models for the 2D and 3D simulations. The rise in the resulting radial outwards pointing radial flux density at the magnet's edge, depending on the distance of the evaluation line or surface to the magnet (compare to Figure 6(b)) is for the 3D case exemplarily shown in Figure 3.

2.2 Estimation of input parameters

In step 2, the input distributions for both error cases were estimated. As an example, the brochure from the magnet manufacturer (Vacuumschmelze GmbH & Co. KG in Hanau, F., 2013) gives error boundaries for the minimal guaranteed magnetization remanence induction in comparison to the typical achieved remanence flux-density. The differences between minimal garanteed and typical magnetic flux-density vary there from 2 to 5 percent depending on the used magnet material, its shape and the applied production process. The measurement data presented in Jurisch (2007) lead, based on their spread, to the assumption, that a Gaussian distribution can be assumed to be a suitable choice for the probability density function (PDF) for the input parameters.

Furthermore, Gaussian distributions are often used in the context of production deviations since a well posed production process should reproduce similar results, with larger deviations being more unlikely than smaller variations.

Both error cases therefore have been chosen to be normal distributed in a way, that the maximum error of the magnetic flux-density's radial component has been allowed to deviate 1.5 percent in positive and negative direction, respectively, defining these points to represent a spread of $\pm 3\sigma$. The corresponding boundaries for both error variables were calculated from this assumption to be $\xi_{A,maxdev} = 1 \pm 0.3$ and $\xi_{B,maxdev} = 1 \pm 0.1$. Both error types were assumed to be independent from each other, resulting in the PDF shown in Figure 4.

2.3 Propagation of uncertainties

For the propagation of uncertainties performed in step 3, Monte-Carlo simulation, based on the described model, has been executed. By using a non-intrusive approach, a polynomial-chaos meta-model (Ghanem and Spanos, 2003) has been created from a subset of the Monte-Carlo samples, following equation (3):

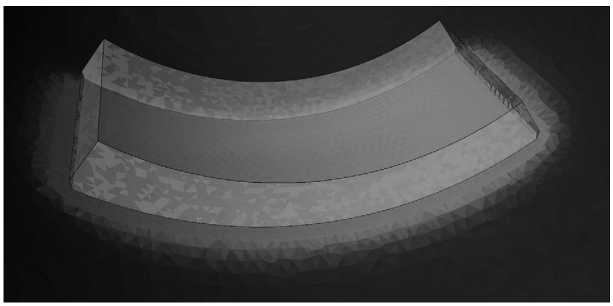


Figure 3.
3D plot of the studied magnet of error type A, showing the flux density's strength on a cylinder parallel to the magnets surface for the ideal radial magnetization

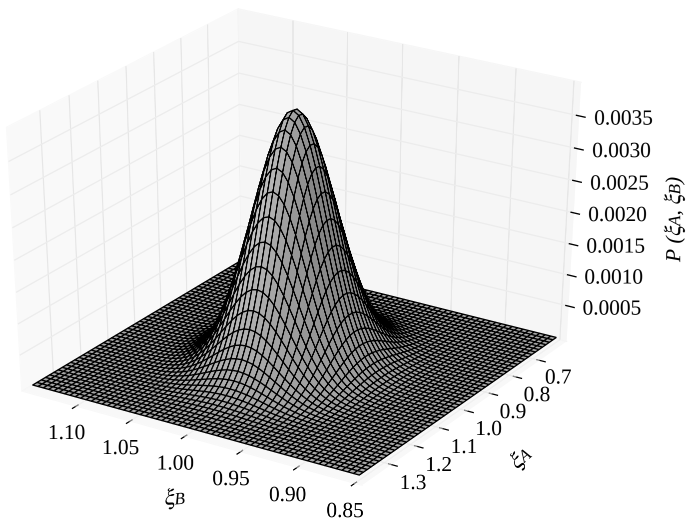


Figure 4.
2D Gaussian PDF of the magnet fault parameters ξ_A and ξ_B used for the Monte-Carlo simulation with $\mu_A = 1.0$, $\mu_B = 1.0$, $\sigma_A = 0.1$ and $\sigma_B = 0.0333$

$$B_r(\omega) = \sum_i a_i \cdot \Phi_i(\xi(\omega)) \tag{3}$$

with ω being an event – meaning one possible realization – in the complete event domain ($\omega \in \Omega$), a_i as polynomial chaos coefficients, Φ_i as polynomial basis and ξ as random vector for the occuring errors. As both errors types are distributed Gaussian in these simulations, the Φ_i are created from Hermite-polynomials:

$$H_n(x) = (-1)^n \cdot e^{x^2/2} \frac{\partial^n}{\partial x^n} e^{-x^2/2} \tag{4}$$

This meta-model (equation (3)) then again was used with Monte-Carlo sampling for the creation of comparable results.

2.4 Output parameter

As output parameter (step 4), the radial component of the flux-density above the magnet has been evaluated on a constant radius at 50 1°-steps above the magnet surface, as shown in Figure 5.

3. Results

Figure 5 shows the resulting field for a magnet of error type B. Since only one sole magnet is considered, the magnet's flux shortens into the same magnet again, leading to higher field densities at the magnet's edge compared to the magnet's center part. This can be observed in figure's five flux lines and the higher flux-density above the magnet's edge on the equidistant cylinder as pictured in Figure 3. Figure 6 shows this evaluation for the ideal, radial magnetized magnet and for a worst case constellation (3σ deviation in both parameters, $\xi_A = 0.7$ $\xi_B = 0.9$) and for a radial magnetized magnet in growing distances. The sharp flux elevation vanished at growing distances

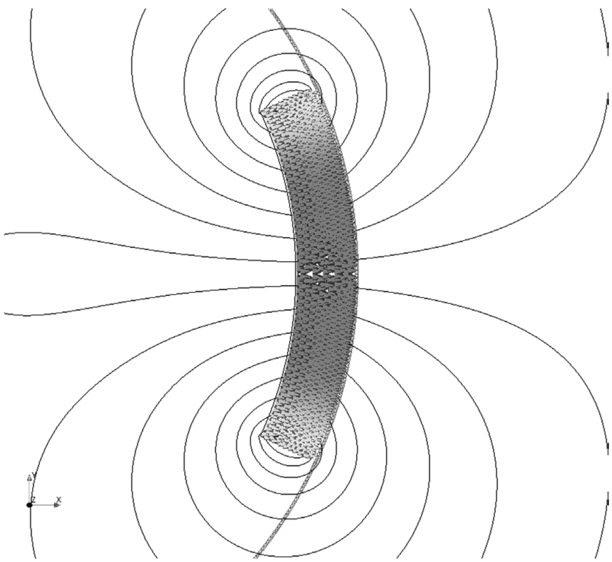


Figure 5.
Field plot of the studied magnet geometry, showing a magnet of error type B

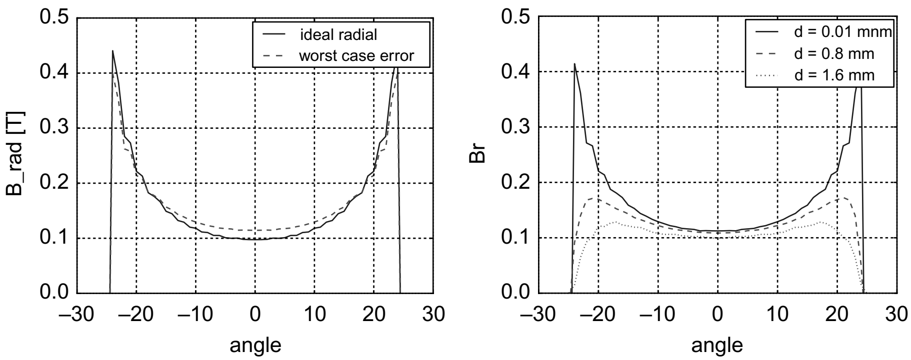


Figure 6.
Radial flux distributions

Notes: (a) Comparison of the radial flux-density along the chosen evaluation line for ideal flux density and worst case flux density (dashed); (b) radial flux-density at different distances for ideal, radial magnetization

as expected relatively fast. However, in order to avoid these distortions the sampling of the CDF has been done at the magnet's center part.

As output size under consideration, the variations of the flux-density at the angle 0 have been simulated with the finite element model and the created polynomial-chaos meta-model. These calculations have been executed as a Monte-Carlo simulation, in order to calculate the flux-density's cumulative density function (CDF). For both techniques, the CDF is shown in Figure 7 and unambiguously describes the probabilistic behaviour of the magnet's flux-density at the considered distance and angle. The graphs of the model and meta-model overlap so smooth, that only for narrow zooms two curves are visible. The applied technique is therefore well suited to be used in a complete machine simulation.

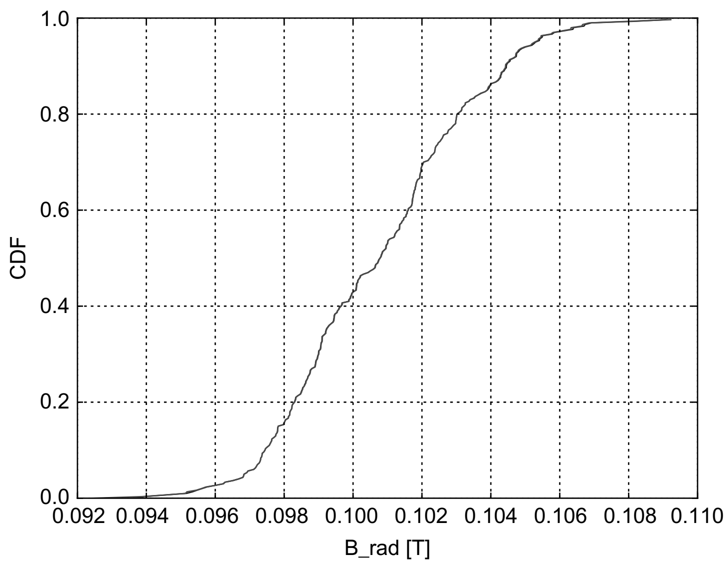


Figure 7.
CDF of the flux-density at the evaluation point at the magnet's center part (angle 0°)

4. Conclusions

The non-linear stochastic variations of two likely magnet errors have been presented along with their influence onto the flux-density above the magnet's surface. It has been verified, that 2D simulations are sufficient in this case. A polynomial-chaos meta-model has been created from simulations for the resulting flux-density along the evaluation line and has been compared to the simulation results themselves.

The meta-model closely fits the simulation data and allows an accelerated simulation, which will be especially useful for larger and more complex models as in 3D. There, the applied variation models however might have to be adopted to include variations in the *z*-axis, too. The meta-model allows the application of sobol indices (Sudret, 2008), which is not necessary in a test case as this magnet, but proves extremely helpful for a sensitivity analysis in a model with large input vectors. Therefore, the considered approach will be used in a next study to estimate the influences of several magnet variations on a rotor onto machine properties as, e.g. induced voltage, cogging torque, etc. The assumed error distribution input functions have to be verified in future work.

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Peter Offermann graduated in electrical engineering at the RWTH Aachen University, Germany, in 2008. Until 2010 he worked at the Institute for Combustion Engines (RWTH Aachen University) in the field of test-bench automation, taking part in the development of a real-time combustion analysis system. He currently is a research associate at the Institute of Electrical Machines of RWTH Aachen University, focusing his work on the propagation of uncertainties across electromagnetic models. His research interests furthermore include numerical field computation, electrical machines, stochastics and optimisation.

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