Influence and evaluation of non-ideal manufacturing process on the cogging torque of a permanent magnet excited synchronous machine

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Abstract

Purpose – The purpose of this paper is to describe how a minimisation of cogging torque is performed with respect to the non-ideal manufacturing process, aiming at a robust design of the studied machine, focusing on magnetisation faults and the use of different qualities of the permanent magnet material.

Design/methodology/approach – The applied methodology is a combination of design of experiments and finite element analysis to minimise the cogging torque of the estimated machine. Different qualities of the permanent magnet material are investigated by a stochastic analysis.

Findings – A robust design of the machine is achieved, which is verified by a stochastic analysis. Furthermore, this analysis shows the strong influence of the magnet quality on the cogging torque.

Practical implications – This paper provides a method for a machine design which is robust against non-ideal manufacturing and an approach to prove the use of a bad quality for a possible reduction of the fabrication costs.

Originality/value – This paper gives a close insight on how to investigate non-ideal manufacturing and in particular its influence on the cogging torque.

Keywords Electrical machines, Torque, Tolerances, Magnetic devices, Manufacturing systems

Paper type Research paper

1. Introduction

In permanent magnet excited machines, cogging torque is caused by the interaction between the rotor magnets and the stator slots. It results in undesired effects such as vibration and deformation. Therefore, a minimisation of such effects is desired.

There are many approaches to achieve a reduction of cogging torque (Islam *et al.*, 2003). All such attempts are usually done for the ideal machine, meaning without considering geometric or material tolerances occurring, e.g. during the fabrication process of the machine or its components. However, it can be shown that such tolerances have a strong influence on the cogging torque (Gasparin *et al.*, 2009) in particular. Ignoring this influence while minimising, results in a machine design which is susceptible to manufacturing faults and tolerances.

This study presents the minimisation of cogging torque for a PMSM considering non-ideal manufacturing process, aiming at a robust design of the machine. Hereby, it is focused on magnetisation faults and the use of different qualities of the permanent magnet material. This study is performed employing a stochastic analysis. An approach to prove the use of a bad quality for a possible reduction of the fabrication costs is provided.

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2. Cogging torque minimisation

2.1 Applied methodology

In this study, the approach to minimize the cogging torque is to optimise the machine's geometry. The applied method is a combination of design of experiments (DoE) (Montgomery, 1974) and finite element analysis (FEA).

DoE is used to identify the significance of the influence of various design parameters concerning the cogging torque for the studied machine. For each numerical experiment, an FE calculation is performed, where the machine is simulated under no-load condition and the cogging torque is computed by the Maxwell stress tensor (Bastos and Sadowski, 2003):

$$T = \frac{l_z}{\mu_0} \int_{\Gamma} r \cdot B_r \cdot B_t \cdot d\Gamma. \tag{1}$$

Here, the radial and tangential component of the magnetic flux density B_r and B_t are integrated along a closed contour Γ around the rotor at radius r. At this, l_z is the length of the machine and μ_0 is the vacuum permeability.

In the first instance, a series of factorial design is performed, whereby the width of the magnet b_M and the width of the slot opening b_S appear to be significant parameters with respect to the cogging torque of the estimated machine. The further optimisation is applied while considering manufacturing tolerances, described in the following.

2.2 Non-ideal manufacturing process

The two considered tolerances of non-ideal manufacturing, which influence the cogging torque and which are known as crucial, are magnetisation faults and a static eccentricity of the rotor (Herranz Gracia, 2008).

In case of tolerances in the magnetisation of the permanent magnets, an asymmetrical distribution of the air gap flux density arises. This results in additional cogging torque. Within this study, it is considered that the magnet's remanence flux density B_R is varying. The amount of additional cogging torque is depending on which and how many magnets are representing a failure. For the studied machine with its eight magnets, 18 configurations can be evaluated as being relevant (Schlensok *et al.*, 2006). Their distribution of cogging torque, in relation to the reference value, is shown in Figure 1, whereby B_R is varying by -5 per cent.

Static eccentricity means that the centre of the rotor is displaced to a fixed eccentric position (le Roux *et al.*, 2003). This is crucial because it results in an asymmetrical distribution of the flux density at the air gap and thereby causes additional cogging torque. In Figure 2, the cogging torque distribution for the 18 configurations with static eccentricity is presented in comparison to the distribution without eccentricity from Figure 1. The two distributions are similar, whereby the one with eccentricity is at higher cogging torque values.

2.3 Results

With the objective of a robust machine design, the optimisation is conducted while considering the above manufacturing tolerances. For each of the 18 configurations, a 2^2-factorial design is applied with b_M and b_S as factors which are varied in two levels. The output value is the peak-to-peak cogging torque ΔT. Table I shows the corresponding matrix of this design. From the resulting coefficients c_i a polynomial can

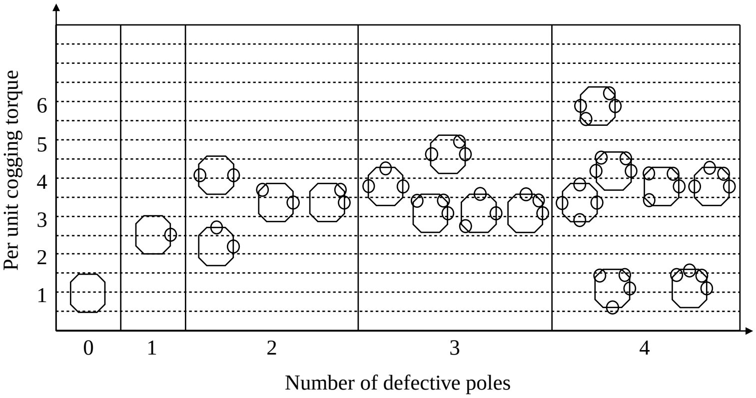


Figure 1.
Cogging torque distribution for the 18 configurations

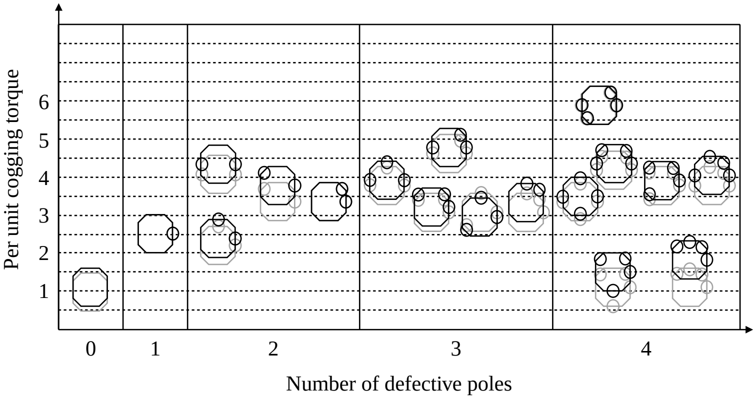


Figure 2.
Cogging torque distribution for the 18 configurations with static eccentricity compared to distribution without static eccentricity

Table I.
2^2-Factorial design

Experiments	b_M	b_S	Independent variables 0	1	2	12	Output
1	−	−	+	−	−	+	ΔT_1
2	+	−	+	+	−	−	ΔT_2
3	−	+	+	−	+	−	ΔT_3
4	+	+	+	+	+	+	ΔT_4
Coefficients			c_0	c_1	c_2	c_{12}	

be determined, which is analysed with the method of gradient descent to finally calculate a minimum for the two design factors.

To obtain an overall optimum for the machine, a statistical evaluation is performed. For each configuration, the probability of occurrence is calculated, assuming the worst case where half of the magnets are representing a failure. Thereby, a weighted average value for the chosen design parameters can be computed, which finally represents the design optimum. For the studied machine, the average of the cogging torque was reduced by 7 per cent when the optimised geometry was applied.

3. Stochastic analysis

3.1 Verification of the optimum

To verify the found design optimum, a stochastic analysis is performed. The remanence flux density B_R is assumed to be normally distributed (Kim *et al.*, 2003). The density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \tag{2}$$

of the normal distribution is shown in Figure 3.

In this study, the variable x is the remanence flux-density B_R with the expected value μ which is set as the reference value of B_R. A tolerance width Δx of 4 per cent is considered, which is equal to the triple standard deviation $\sigma = \Delta x/3$. By use of the Box-Muller method (Box and Muller, 1958), 50 random configurations are created with a random value of B_R for each magnet. The cogging torque is computed for each configuration, with reference as well as with optimised geometry.

The results are divided into 16 intervals, where interval I is the one with the lowest and XVI the one with the highest values of cogging torque. Figure 4 shows the resulting frequency distribution for both geometries. The distribution for the optimised geometry is shifted to lower intervals compared to the reference one, which shows the achieved reduction of cogging torque. On average, the cogging torque is 9 per cent lower when compared to the model with reference geometry.

3.2 Consideration of magnet quality

The quality of the permanent magnets depends on the manufacturing process. A bad quality means that either the tolerance width of B_R or the failure probability is higher. Both cases are studied in the following.

The same procedure as described in Section 3.1 is performed for a tolerance width Δx of 6 per cent. Figure 5 shows the corresponding frequency distribution. The intervals are identical to those in Figure 4. Applying the optimised design for

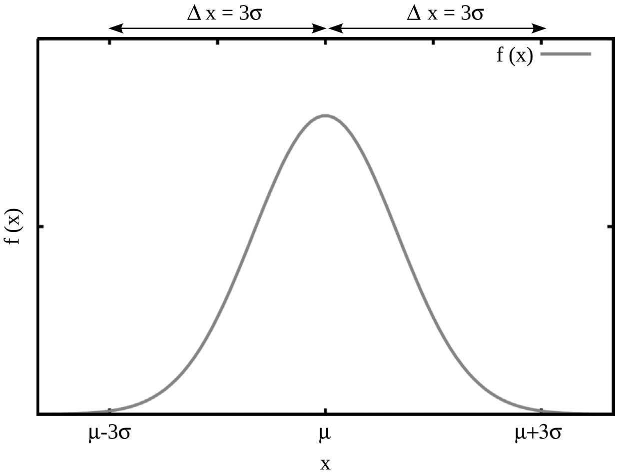


Figure 3.
Density function of the normal distribution

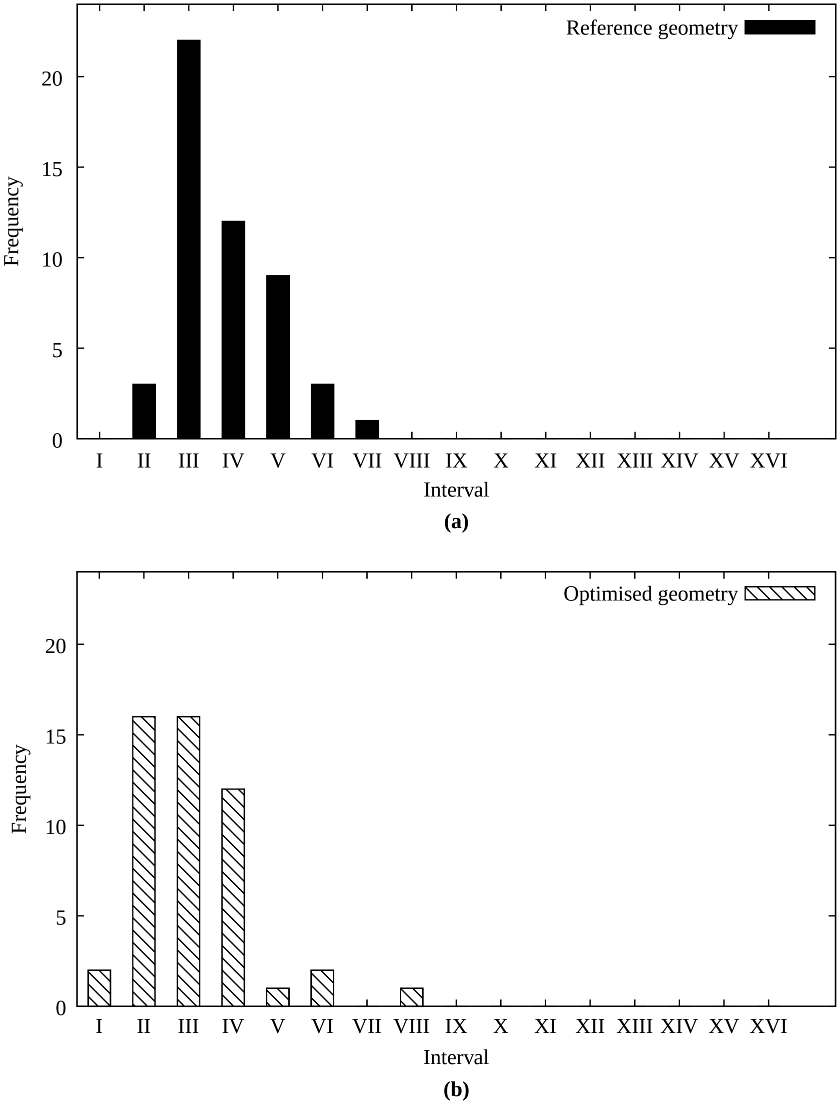
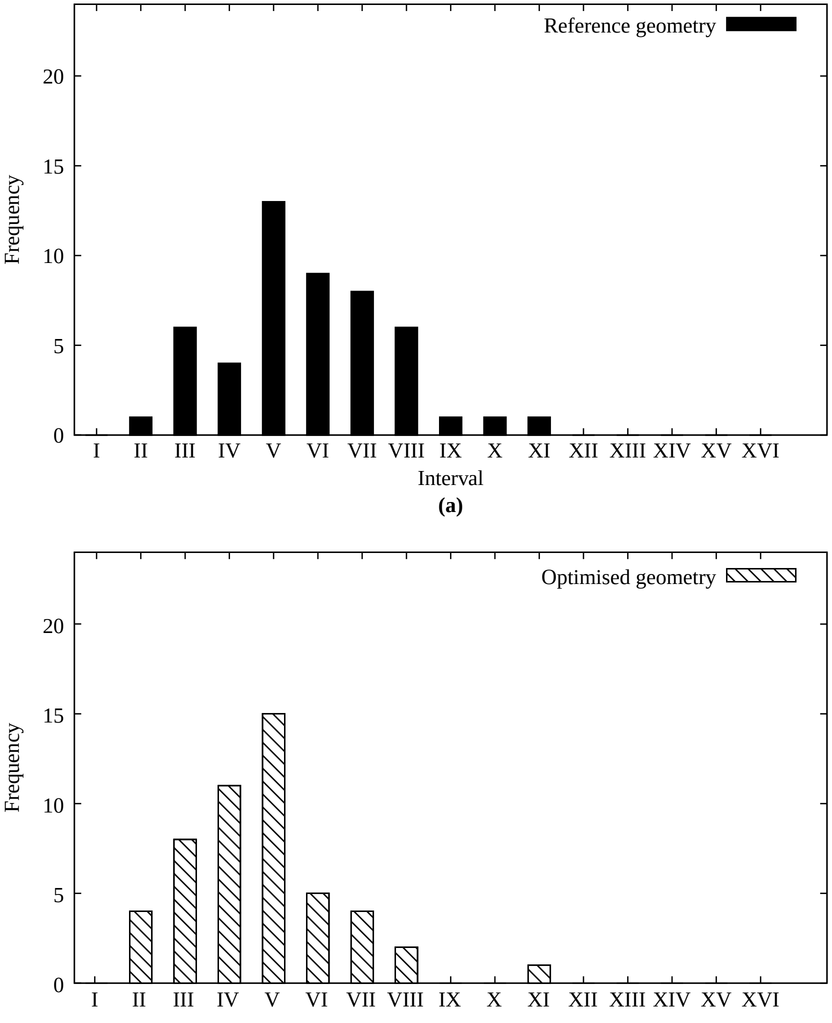


Figure 4.
Frequency distribution – normally distributed tolerances with 4 per cent tolerance width

Notes: (a) Reference geometry; (b) optimised geometry

this case, leads on average to 10 per cent less cogging torque. Compared to the results of the distribution from Section 3.1, the variance of distribution and the average values of cogging torque are higher.

When assuming a uniform distribution for the magnetisation, all failures appear with the same probability. This is inferior to the normal distribution where the density function is a bell-shaped curve. The density function of a uniform distribution is described by:



(a)

(b)

Notes: (a) Reference geometry; (b) optimised geometry

Figure 5.
Frequency distribution – normally distributed tolerances with 6 per cent tolerance width

$$f(x) = \begin{cases} 0 & x < a \\ \dfrac{1}{b-a} & a \leq x \leq b \\ 0 & x > b \end{cases}. \tag{3}$$

Here, the expected value μ is equal to the reference value of the remanence flux density B_{Rref}. A tolerance width of 4 per cent is assumed, meaning that the parameter a is at a variance of -4 per cent and b is at a variance of $+4$ per cent of B_{Rref}. The same procedure as described for the normal distributed tolerances is performed. Figure 6 shows the resulting frequency distribution for reference and optimised geometry of the model. The average reduction of cogging torque, by applying the optimised geometry, is 9 per cent. Compared to the results of both normal distributions, the variance of distribution and the average values of cogging torque are higher.

3.3 Results

Finally, the results of this stochastic analysis prove that the optimised geometry is robust against manufacturing faults. The influence of varying qualities of the magnets

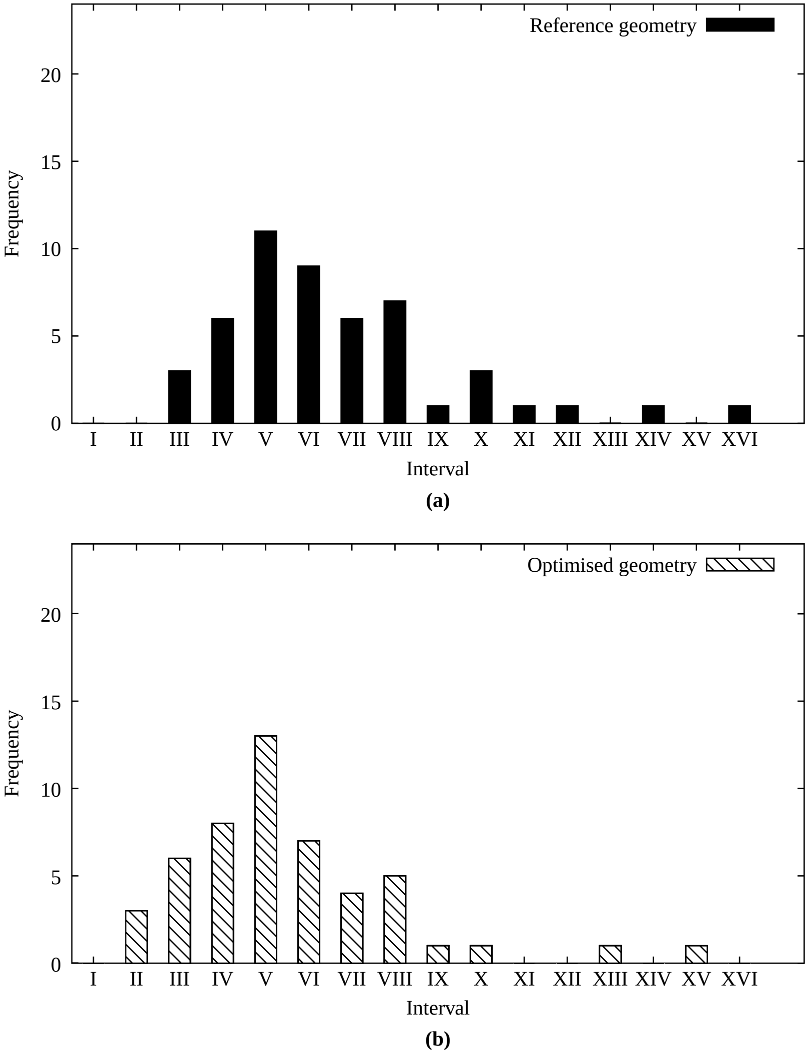


Figure 6.
Frequency distribution – uniformly distributed tolerances with 4 per cent tolerance width

Notes: (a) Reference geometry; (b) optimised geometry is shown by modelling the tolerances using different probability distributions. The three considered tolerance distributions show the same percental amount of reduction in the average cogging torque. Whereas, the values of cogging torque and their distribution is strongly influenced by the assumed probability distribution. From the three considered cases, the uniform distribution presents the worst case.

In general, it can be stated that a better quality of the magnets requires a higher precision during manufacturing, which may result in higher material costs. This study provides an approach to prove the use of a bad quality for a possible reduction of the fabrication costs.

4. Conclusion

This study shows the minimisation of cogging torque of a PMSM with respect to manufacturing tolerances and failures. A robust design of the machine is achieved by using numerical simulations combined with statistical methods such as DoE. A stochastic FEA is performed to validate this design and to investigate the influence of different qualities of the permanent magnets. It has been shown that the magnet quality has a strong influence on the cogging torque of the estimated machine. Within this scope, an approach for a possible reduction of the fabrication costs is provided.

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