

## CALCULATION OF END-WINDING FORCES OF INVERTER FED DRIVES

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**Abstract** – End-winding forces can be approximated applying the Biot-Savart law. Usually constant speed and ideal sinusoidal currents are assumed. Applying this constrains a simulation of the forces for one electrical period is sufficient. However, for inverter fed drives, the harmonic content of the current is dependent on the operational load and speed. In this case the forces for each time step of the drive cycle of traction drive have to be calculated individually. This paper proposes a superposition of so called unit-current forces for each current harmonic. Still, the forces are calculated for one electrical period with unit-current amplitude based on Biot-Savart's law. The calculation of the resulting forces is performed by means of a current superposition. This allows for a computational cost effective simulation of end-winding forces taking current harmonics due to inverter supply, speed and load dynamic operations into account.

### Introduction

End-winding forces can mechanically damage the winding system. The development of high power converters yields to their application in large drives, where end-winding forces are relevant. In this applications speed and load dynamic operations are required. The authors found that the influence of current harmonics on the end-winding forces is significant. Purpose of the research is the evaluation of end-winding forces in dynamic operation. Therefore, 3D-FEM yields to unacceptable computational efforts. The classical Biot-Savart approach assuming linear material properties allows for a good approximation of these forces [1]. In this paper a computational efficient procedure to calculate the end-winding forces is presented, which is based on Biot-Savart's law. Basis of the presented approach is the force calculation for one electrical period of a sinusoidal current with unit amplitude. A superposition of all current harmonics is proposed. The force calculation is then performed in frequency domain by means of convolutions.

### Superposition of Unit Forces

The electromechanical forces are calculated analytically based on a line conductor fragment model of the end-winding geometry using Biot-Savart's law and Lorentz forces [1-2]. To obtain a unit force distribution, the forces are calculated once with the Biot-Savart law for one electrical rotation of the machine with a sinusoidal current of given amplitude  $\hat{I}$  and frequency  $f_t$ . A Fourier transformation yields the amplitudes  $\hat{a}$  of the harmonic force waves with fundamental frequency  $f_0$

$$a_p(t) = \sum_{k=0}^{K-1} \hat{a}_{p,k} \cdot \cos(k \cdot 2\pi f_0 t - \varphi_{a,k}). \quad (1)$$

The appearing frequencies of the force waves arise from the multiplication of the current signal in the time domain which corresponds to a convolution in the frequency domain

$$\mathcal{L}\{i[n] \cdot i[n]\} = I[r] * I[r] = \frac{1}{N} \sum_{s=0}^{N-1} I[s] \cdot I[r-s]. \quad (2)$$

Since a linear time invariant model is used, the resulting forces can be normalized to the amplitude of the given current. With

$$F \sim I^2 \quad (3)$$

the unit force  $a_{e,k,P_q}$  in point  $P_q$  is consequently determined by

$$a_{e,k,P_q} = \frac{\hat{a}_k}{(\hat{i} / \sqrt{2})^2}. \quad (4)$$

The electromagnetic forces for various current patterns are determined using weighted superposition of unit forces, taking into account the phasing of the currents' frequency components. As in (1) the Fourier transformation of the squared current signal yields

$$i^2(t) = \sum_{m=0}^{M-1} \hat{i}_m \cdot \cos(m \cdot 2\pi f_s t - \varphi_{i,m}), \quad (5)$$

with fundamental frequency  $f_s$  and phase angle  $\varphi_{i,m}$ . The electromagnetic force  $F_{P_q}(t)$  in one point  $P_q$  caused by a current  $i(t)$  is then given by the convolution

$$F_{P_q}(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} a_{e,k,P_q} \cdot \hat{i}_m \cdot \cos(m \cdot 2\pi f_s t - (\varphi_{\alpha,k} + \varphi_{i,m})). \quad (6)$$

This procedure has to be repeated for each point in the model, where forces are to be calculated.

## Results

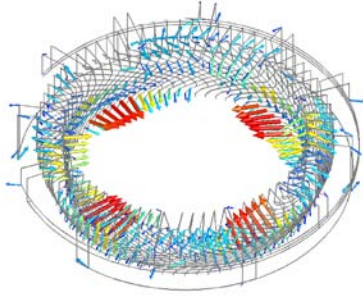


Fig. 1 Forces calculated in 3D-modell.

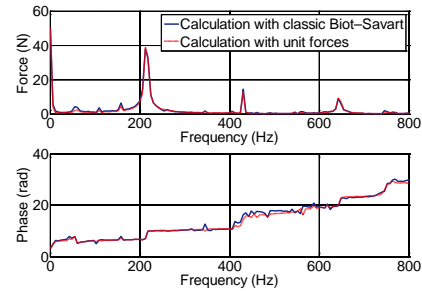


Fig. 2 Comparison of the calculated forces.

As an example the electromagnetic forces acting on the end-winding are calculated based on the classical Biot-Savart and the unit-current force superposition approach for a constant load and speed, taking into account current harmonics due to the inverter. Fig. 1 shows the obtained force distribution. In order to evaluate the unit-current force approach, the calculated forces at one point with both approaches are shown in Fig. 2. Both approaches show a good agreement in amplitude and phase.

## Summary

A unit-current force-superposition method is proposed as an easy to handle, time-efficient method to determine the electromagnetic forces on the end-winding of electrical drives. Linear material properties are assumed. The presented approach allows for the time efficient calculation the end-winding forces for dynamic operation of the machine taking various current patterns into account. It is found that even small amplitudes of current harmonics can result in large force amplitudes. In the full paper the variation of the force in time for a ramp-up of the machine will be presented. A mapping of the generated forces and the exciting current harmonics will be presented.

## References

- [1] Drubel O., Kulig S., Senske K.: *End Winding Deformations in Different Turbo Generators During 3-Phase Short Circuit and Full Load Operation*, *Electrical Engineering*, Springer Verlag, vol. 82, pp. 145-152, 2000.
- [2] C.J. Carpenter, *The application of the method of images to machine end-winding fields*, *Proceedings IEE Part A*, Vol.107, pp.487-500, 1960.